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EFFECTS OF BROWNIAN MOTION ON THE MILLIKAN OIL DROP EXPERIMENT

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Brownian motion has a significant effect on small particles suspended in a fluid. Since the Millikan oil drop experiment involves measuring the rise and fall velocities of very small oil drops suspended in air, it stands to reason that the motion of these drops will be affected by collisions with particles of air. The result of this is that the measured rise and fall velocities of each drop will not be the same as if these drops were suspended in vacuum. The size of the effect of Brownian motion is related to the mass and the radius of the oil drop, and is also related to the temperature of the surrounding fluid. In our experiment, we calculated the charge of multiple drops of varying size. The charge on each was first calculated without considering Brownian motion, and then was calculated again while taking Brownian motion into account. When Brownian motion was taken into consideration, the accuracy of our calculation of the fundamental charge of an electron improved by a factor of six, permitting a determination of the value of the fundamental unit of electric charge (the charge of an electron) that differed from the accepted value by less than 1%. The effect of Brownian motion in the classic Millikan oil drop experiment is significant enough to be observed with basic undergraduate apparatus if sufficient precision is attained by careful measurement.

1 Introduction

The fundamental charge of an electron was first measured by Robert Millikan in 1909. He also discovered that the charges found in each drop were multiples of charge, leading to the quantization of charge. Millikan received the Nobel Prize in Physics in 1923 for his experiment. The accepted fundamental charge of an electron, $|e|$, is $1.602176462 \times 10^{-19}$ Coulombs (C) [1]. A Coulomb is the amount of charge that is carried in a one Ampere current for 1 second.

The principle of operation for the Millikan oil drop experiment is that the charge of an oil drop with one or a few excess electrons can be found by measuring the rise time of the drop in an electric field of known strength and the free fall time with no electric field. This is achieved by spraying small drops of oil between a parallel plate capacitor with a known voltage potential applied across the plates.

The oil drops are small enough to be affected by Brownian motion. Brownian motion occurs when air molecules interact with the oil drops. Brownian motion causes the drop to fall in a random manner. Instead of a straight path of descent or ascent, the oil drops undergo a “random walk”. Thus, the oil drop is actually traveling a greater distance than the straight line path; therefore, a correction in distance traveled was made using Einstein’s equation for Brownian motion.

The purpose of this experiment was to measure the fundamental charge of an electron with its associated uncertainty considering Brownian motion. Since some drops may have more than one electron, this experiment will also support the claim that charge is quantized for cases of charge greater than one *e*.

2 Experiment

In this section the apparatus, procedure, and data will be discussed.

2.1 Apparatus

The apparatus used for this experiment was the PASCO© scientific Millikan Oil Drop Apparatus, Model AP-8210. This apparatus was designed to find the fundamental charge of an electron and to prove that the charge is quantized. The experiment was conducted at Minnesota State University in Mankato, Minnesota in the Trafton Science Center room TR-C108.

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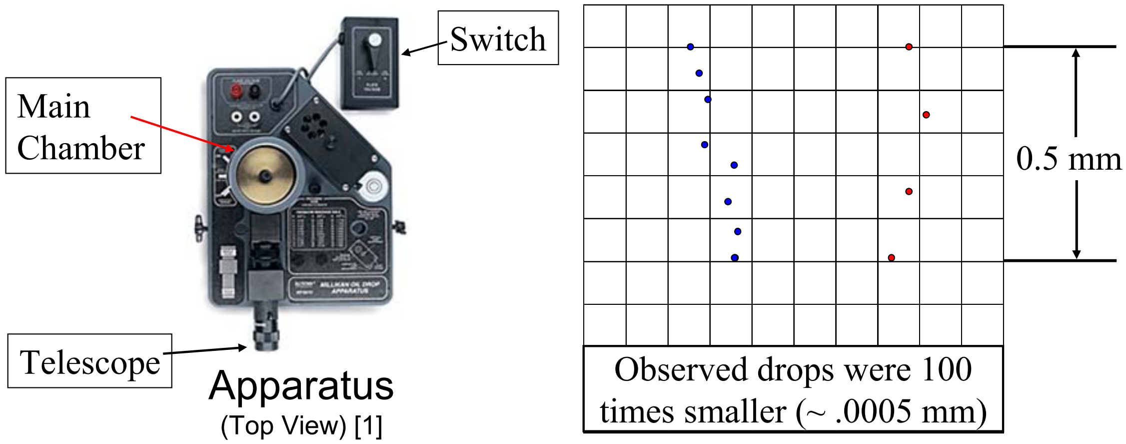

Figure 1: (a) PASCO© Millikan Oil Drop Apparatus, Model AP-8210 [2]. (b) Representation of the oil drops as seen in the apparatus viewing chamber.

The charging of the capacitor plates created the electric field necessary to conduct the experiment. The charging of the plates was controlled by a switch that could turn the field off or charge the field with two polarizations so the oil drop could be induced to rise with the field or fall with the field. A viewing scope was placed between the plates so that the oil drops could be observed. The telescope had reticle marks spaced to 0.1mm and major reticle marks at 0.5mm so that rising and falling times could be observed. Figure 1(b) shows oil drops falling (blue) and rising (red). Each drop represents the position of the oil drop in a two second time interval.

A thermistor was used to determine the temperature in the viewing chamber between charging plates. The thermistor was connected to a digital multimeter (DMM) to measure the resistance. The measured resistance was then used to determine the temperature of the viewing chamber.

2.2 Procedure

The power supply connected to the capacitor was then turned on, along with a Hewlett-Packard DMM connected across the capacitor. The power supply was set at approximately 500 Volts. The drops were viewed through the viewing scope and adjustments were made to the scope until the majority of the drops could be seen clearly. The field was then turned on so that the drops would rise. The field was turned on and off until a desirable drop was found. Experience helped determine which drops were desirable. The rise and fall times were recorded for a set distance using the timer. This procedure was repeated for a total of seven drops.

2.3 Data

Seven data sets were recorded. Typical values for fall velocity (without electric field) and rise velocity (with electric field) are shown in Table 1:

Trial Results						Ave (s)	+/-
t_f (s)	14.54	14.19	14.34	14.19	14.10	14.28	0.08
t_r (s)	3.05	3.11	3.28	3.44	3.52	3.28	0.09

Table 1: Typical oil drop fall and rise times.

The temperature of the viewing chamber was obtained from the resistance of the thermistor reading from the DMM connected to it. The resistance was then compared to the linear approximation plot of thermistor resistance versus temperature localized for temperatures between 293-298 Kelvin (K). The viscosity of air in poise (η) was obtained from the temperature value described above. The barometric pressure (p) was obtained from the digital weather station located in the showcase outside of Trafton TR-N159. The sensor is located on the roof of the Trafton Science Center. The voltage (V) between plates was recorded from the DMM which was connected in parallel with the power supply to the apparatus. The separation between the plates (d) was measured, using a micrometer, to be (0.007676 ± 0.000005) m. The density of the oil (ρ) was determined to be (876.1 ± 18.5) $\frac{\text{kg}}{\text{m}^3}$. Typical values for the above variables are shown in Table 2.

Variable	Definition	Units	Value	+/-
d	Separation of Plates	m	7.67×10^{-3}	0.05×10^{-3}
ρ	Oil Density	kg/m^3	876.1	18.5
g	Acceleration due to gravity	m/s^2	9.81	0.01
η	Viscosity of Air	Ns/m^2	1.837×10^{-5}	0.001×10^{-5}
b	Constant	Pa m	8.20×10^{-3}	N.A.
p	Air Pressure	Pa	1.01388×10^{5}	0.00034×10^{5}
t_f	Fall Time	s	14.28	0.08
t_r	Rise Time	s	3.28	0.09
V	Voltage Across Plates	V	500	1

Table 2: Typical values for variables considered in the calculation of the charge (*q*).

The variables above were used in the determination of the excess charge on each drop and the Brownian motion correction.

3 Analysis

Analysis of the oil drop radius, mass, charge, and effects of Brownian motion will be discussed.

3.1 Radius, Mass, and Charge

The radius (r) of the oil drop was calculated using:

$$r = \left[\sqrt{\left(\frac{b}{2p}\right)^2 + \frac{9\eta z}{2gt_f\rho}} - \frac{b}{2p}\right]$$

The variable z in the above equation is the distance the oil drops were timed for, which was 0.5mm. The mass (m) of each drop can be found using the equation $m = \frac{4}{3}r^3\pi\rho$.

Substituting for r, the mass can be found to be:

$$m = \frac{4}{3}\pi\rho g\left[\sqrt{\left(\frac{b}{2p}\right)^2 + \frac{9\eta z}{2gt_f\rho}} - \frac{b}{2p}\right]^3$$

The charge (q) on the oil drop can then be expressed by:

$$q = mg\left(1 + \frac{t_r}{t_f}\right)\frac{d}{V}$$

After substituting for m, the charge can finally be expressed as:

$$q = \left[\sqrt{\left(\frac{b}{2p}\right)^2 + \frac{9\eta z}{2gt_f\rho}} - \frac{b}{2p}\right]^3\left(1 + \frac{t_r}{t_f}\right)\frac{d}{V}$$

The value for b is a constant, 8.20×10^{-3} Pa m. The values for the other variables can be found in Table 2.

The uncertainty in q written here as δq was determined using the error propagation equation below. It was derived by taking the partial derivatives of the associated terms.

$$\left(\frac{\delta q}{q}\right)^2 = \left(\frac{\delta\rho}{\rho}\right)^2\left(\frac{\rho}{q}\right)^2\left(\frac{\partial q}{\partial\rho}\right)^2 + \left(\frac{\delta g}{g}\right)^2\left(\frac{g}{q}\right)^2\left(\frac{\partial q}{\partial g}\right)^2 + \left(\frac{\delta p}{p}\right)^2\left(\frac{p}{q}\right)^2\left(\frac{\partial q}{\partial p}\right)^2 + \left(\frac{\delta\eta}{\eta}\right)^2\left(\frac{\eta}{q}\right)^2\left(\frac{\partial q}{\partial\eta}\right)^2$$
$$+ \left(\frac{\delta t_f}{t_f}\right)^2\left(\frac{t_f}{q}\right)^2\left(\frac{\partial q}{\partial t_f}\right)^2 + \left(\frac{\delta t_r}{t_r}\right)^2\left(\frac{t_r}{q}\right)^2\left(\frac{\partial q}{\partial t_r}\right)^2 + \left(\frac{\delta V}{V}\right)^2\left(\frac{V}{q}\right)^2\left(\frac{\partial q}{\partial V}\right)^2 + \left(\frac{\delta d}{d}\right)^2\left(\frac{d}{q}\right)^2\left(\frac{\partial q}{\partial d}\right)^2$$

The partial terms in the uncertainty calculation were evaluated using a numerical approximation. The numerical approximation can be described by the equation below:

$$\frac{\partial f(x)}{\partial x} = \frac{\left(f(x+c) - f(x-c)\right)}{2c}$$

where c<<x.

The greatest contributions to the uncertainty of charge were from the density of the oil, fall time, and rise time, respectively. The density of the oil was determined using Archimedes's Principle where the specific gravity and the known density of water were used to find the oil density.

3.2 Brownian Motion

Brownian motion is caused by collisions of fluid (air) molecules with an object (oil drop). The oil drops are small enough to be noticeably affected by these collisions. The "random walk" of the oil drops can be attributed to Brownian motion. A simplified "random walk" is shown in Figure 2:

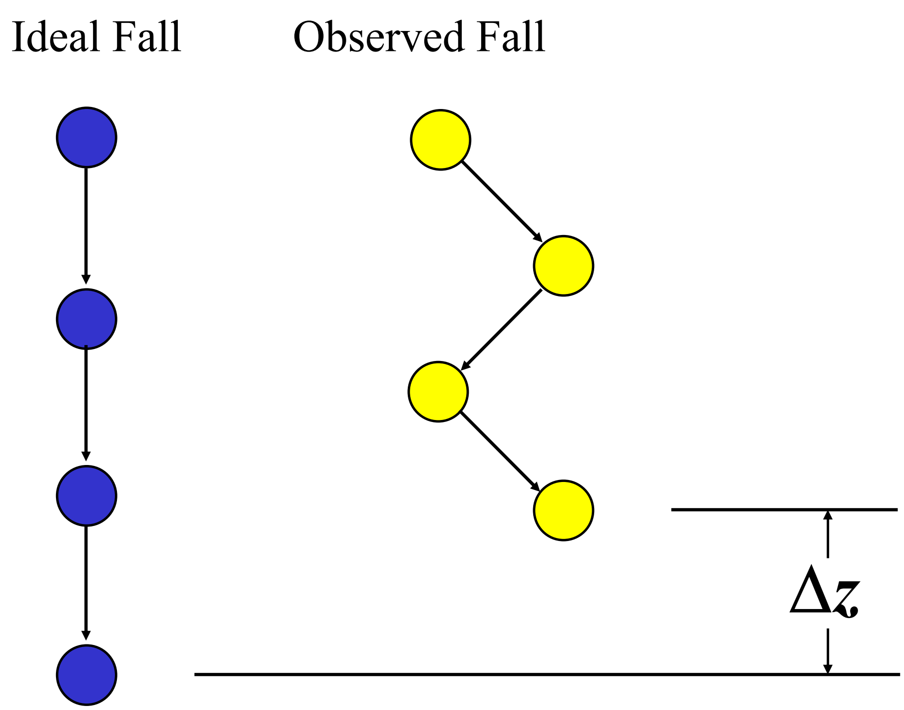

Figure 2: Simplified diagram of Brownian motion acting on an oil drop.

Both drops travel an equal total distance in the same time interval. The dots are separated by arrows of equal length to represent the positions of the drop during a given time interval. Notice that in both cases the same number of arrows is used. Thus, the total path length for the observed drop was greater than the vertical distance traveled. Therefore, the distance used in the determination of q was not the correct distance traveled for the recorded time. This explained the consistently lower experimental values of the electron charge compared to the accepted value. The actual distance traveled by the drop was found by adding the following correction term to the measured distance:

$$\Delta z = \sqrt{\frac{2k_B T}{6\pi\eta r}\Delta t}$$

where r is the radius of the drop, η is the viscosity of air in poise, Δt is the time interval considered, k_B is Boltzman's Constant, and T is the absolute temperature in the viewing chamber [3].

4 Results

The charge on each oil drop was calculated for the uncorrected and corrected cases. The values are listed in Table 3:

Drop Number	Uncorrected Charge (10^{-19} C)	Corrected Charge (10^{-19} C)
1	2.934 ± 0.062	3.131 ± 0.066
2	1.476 ± 0.032	1.600 ± 0.035
3	2.907 ± 0.076	3.207 ± 0.085
4	4.668 ± 0.110	4.829 ± 0.114
5	7.641 ± 0.136	7.877 ± 0.140
6	7.773 ± 0.127	8.007 ± 0.131
7	6.290 ± 0.099	6.488 ± 0.102

Table 3: Uncorrected and corrected charges for each oil drop along with the associated uncertainties.

The above data is discussed in the next section.

5 Discussion

The uncorrected charges ranged from 1.476×10^{-19} C to 7.773×10^{-19} C and the corrected charges ranged from 1.609×10^{-19} C to 8.047×10^{-19} C. To find the number of electrons on each oil drop, the charges were divided by the fundamental charge, $e = 1.602\times10^{-19}$ C. The results for both uncorrected and corrected cases are shown in Table 4 below.

Drop Number	Ratio of Uncorrected Charge to ***e***	% Difference	Ratio of Corrected Charge to ***e***	% Difference
1	1.831	8.45	1.954	2.30
2	0.921	7.90	0.999	0.10
3	1.814	9.30	2.002	0.10
4	2.914	2.87	3.014	0.47
5	4.770	4.60	4.917	1.66
6	4.852	2.96	4.998	0.04
7	3.926	1.85	4.050	1.25

Table 4: The ratio of oil drop charge to fundamental charge and the percent difference of the ratio from the nearest integer.

Above, the values of the ratio of oil drop charges to the fundamental charge are all nearly integer values. This supports the theory of quantization of charge. If the ratio were fractional or not nearly integer values, this would contradict the theory of quantization of charge. The largest deviation from the accepted value of *e* was in the charge found on the third drop. The percent difference from *e* was 9.30% for the uncorrected charge. The percent difference between the corrected value and *e* for the third drop was 0.10% (the charge was slightly larger than the accepted value). The greatest percent difference for the corrected charge values was for the first drop (2.30%) which is still in very good agreement with the accepted value.

The corrected values for the charge gave the best support for quantized charge. Notice that the corrected values give the least percent difference from the accepted value, which follows the trend of the corrected charges having the highest accuracy.

Using the above data, the fundamental charge of an electron was calculated from the weighted average of the total charge divided by N, which was determined above.

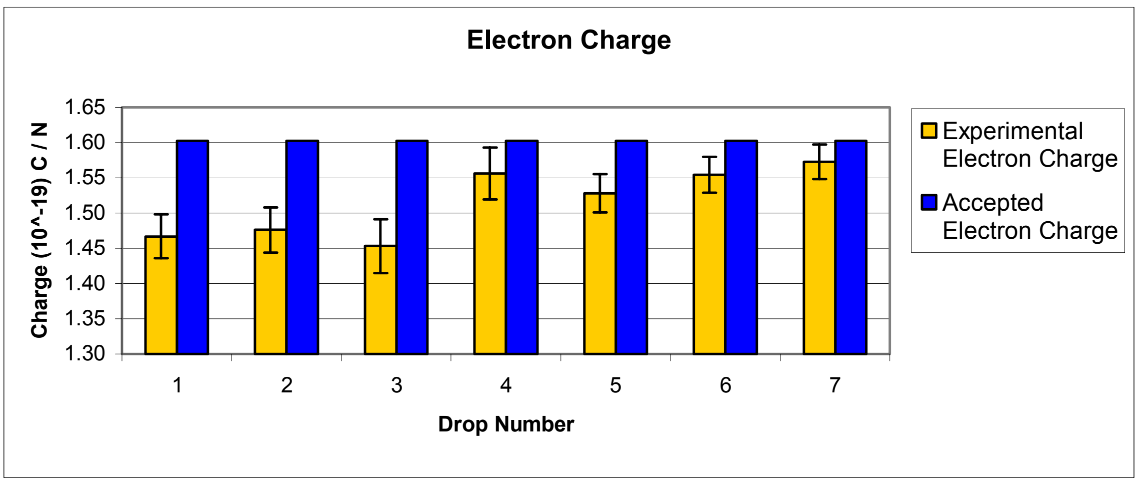

Figure 3: Uncorrected fundamental charges determined from each drop.

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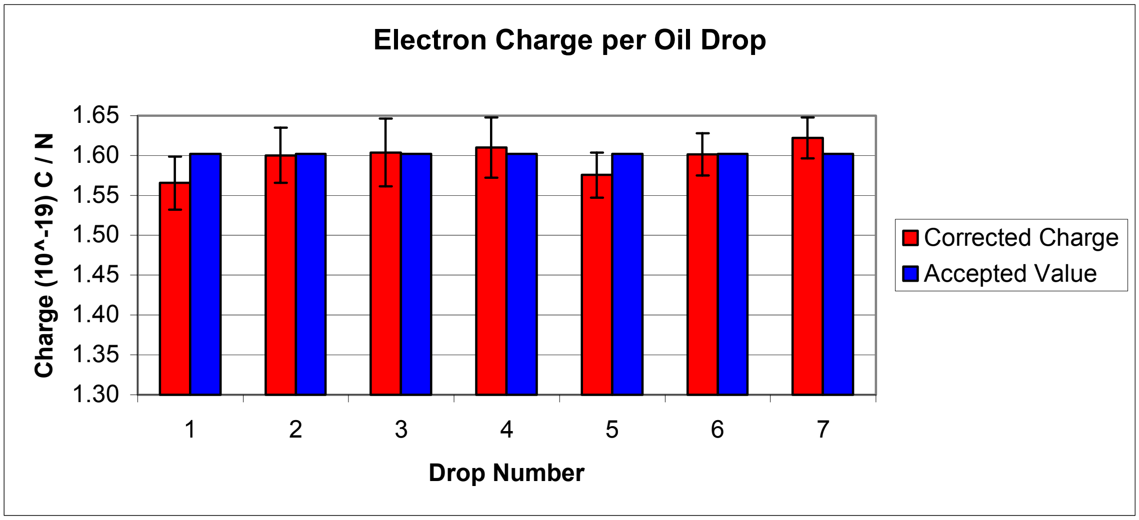

Figure 4: Corrected fundamental charges determined from each drop.

The uncorrected consideration produced a result of $|e| = 1.531\times10^{-19}$ C. The percent difference from the accepted value was 4.43%. The corrected charges resulted in $|e| = 1.597\times10^{-19}$ C. The associated percent difference from the accepted value was 0.31%. In this experiment, the correction factor improved the accuracy by nearly a factor of ten. The calculated charge of an electron agrees extremely well with the accepted value when the correction term is taken into consideration.

6 Conclusion

The fundamental charge of an electron was originally calculated without considering Brownian motion. Later, Brownian motion was considered in the calculation of the fundamental electron charge.

Original Charge Calculation:

$$q = (1.531 \pm 0.029)\times10^{-19}\ \mathrm{C}$$

Corrected Charge Calculation:

$$q = (1.597 \pm 0.031)\times10^{-19}\ \mathrm{C}$$

The experimental results are in agreement with the accepted value of the fundamental electron charge e. The calculated values of q are within one standard deviation of the accepted value. The uncertainties for the charges on each drop are to the order of 10^{-20} C.

The major source of error for each case was the density of the oil drops. The uncertainty of the rising velocity with electric field on was a little larger than the uncertainty of the density of the oil in some cases, but the density of the oil was consistently one of the leading contributors. Precision in falling and rising time could be increased with a greater number of measurements, but Brownian motion will cause the uncertainty to remain on the same order of magnitude since the motion of the drops is somewhat random.

This experiment shows that relatively accurate measurements of the fundamental charge ($|e|$) can be made using a simple apparatus. Not only can accurate measurements of $|e|$ be made with the basic apparatus, but the effects of Brownian motion can be observed and integrated into the determination of $|e|$. If great care is put into the measurements in order to maintain a high order of precision, the correction for Brownian motion can significantly change the original calculation of the fundamental charge.

Further investigation into the effect of the evaporation of the oil drops on the results of the Millikan Oil Drop Experiment would be interesting and are very likely to be the next most significant effect on the accuracy of the experiment. The effects of a non-uniform electric field between the capacitor plates would be another possible area of research.

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7 Bibliography

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8 Author Biographies

Authors' Biographies

Eric Ehler is a graduate of the physics department at Minnesota State University, Mankato. He will be attending Medical School in the University of Wisconson's Medical Physics program with a full Research Assistantship. His initial research will be in the fMRI (functional MRI) research group at the University of Wisconson.

Aaron Hanson is currently a senior at Minnesota State University, Mankato. He will graduate in Fall 2005 with degrees in math and physics. He plans on going to graduate school for physics upon graduation from MSU.

Faculty Mentor Biography

Mark Pickar is a professor in the Department of Physics and Astronomy at Minnesota State University, Mankato. He joined the faculty of this department in 1997, and has been the chair of the department since the fall of 2003. He is an experimental nuclear physicist whose current interests include studies of symmetry breaking forces in few nucleon systems, computer simulation methods to model and analyze experiments in nuclear physics, nucleon induced pion production, and high energy photon production in medium energy nuclear collisions.