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THE IMPACT OF NON-EQUIPARTITION ON COSMOLOGICAL PARAMETER ESTIMATION FROM SUNYAEV–ZEL'DOVICH SURVEYS

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ABSTRACT

The collisionless accretion shock at the outer boundary of a galaxy cluster should primarily heat the ions instead of electrons since they carry most of the kinetic energy of the infalling gas. Near the accretion shock, the density of the intracluster medium is very low and the Coulomb collisional timescale is longer than the accretion timescale. Electrons and ions may not achieve equipartition in these regions. Numerical simulations have shown that the Sunyaev–Zel'dovich observables (e.g., the integrated Comptonization parameter Y) for relaxed clusters can be biased by a few percent. The Y versus mass relation can be biased if non-equipartition effects are not properly taken into account. Using a set of hydrodynamical simulations we have developed, we have calculated three potential systematic biases in the Y versus mass relations introduced by non-equipartition effects during the cross-calibration or self-calibration when using the galaxy cluster abundance technique to constraint cosmological parameters. We then use a semi-analytic technique to estimate the non-equipartition effects on the distribution functions of Y (Y functions) determined from the extended Press–Schechter theory. Depending on the calibration method, we find that non-equipartition effects can induce systematic biases on the Y functions, and the values of the cosmological parameters Ω_8, σ_8, and the dark energy equation of state parameter w can be biased by a few percent. In particular, non-equipartition effects can introduce an apparent evolution in w of a few percent in all of the systematic cases we considered. Techniques are suggested to take into account the non-equipartition effect empirically when using the cluster abundance technique to study precision cosmology. We conclude that systematic uncertainties in the Y versus mass relation of even a few percent can introduce a comparable level of biases in cosmological parameter measurements.

Key words: cosmic background radiation – cosmological parameters – galaxies: clusters: general – hydrodynamics – intergalactic medium – large-scale structure of universe

1. INTRODUCTION

Observational and theoretical studies have shown that galaxy clusters can be used as cosmological probes. In particular, the evolution of the galaxy cluster abundance, or the mass function, is sensitive to cosmological parameters including the average matter density Ω_M, the normalization of the power spectrum of the initial density fluctuations σ_8, and the dark energy equation of state parameter w (Mantz et al. 2008; Vikhlinin et al. 2009). Except for gravitational lensing which is difficult to do for large sample of galaxy clusters, the masses of galaxy clusters cannot be directly measured, and hence the mass function cannot be measured easily. Very often, the masses are estimated using mass proxies such as the Sunyaev–Zel'dovich (SZ) temperature distortion, the X-ray flux, or galaxy dynamics. The mass–observable relations have to be calibrated empirically or semi-empirically with numerical simulations (e.g., Vikhlinin et al. 2009). Hence, measuring cosmological parameters using the galaxy cluster abundance requires a full understanding of mass–observable relations. Even if the mass function and the mass–observable relation can be fitted simultaneously ("self-calibration"; Levine et al. 2002; Hu 2003; Majumdar & Mohr 2003), the correct form of the mass–observable relation is needed.

A recent X-ray survey has shown that even a sample of only 85 X-ray clusters is sufficient to provide very tight constraints on some cosmological parameters. For example, σ_8 can be measured down to a 1% level in statistical uncertainty using the cluster abundance technique alone by assuming a flat universe with fixed dark energy equation of state parameter and a prior on the Hubble constant (Vikhlinin et al. 2009). However, the statistical uncertainties on some other cosmological parameters (e.g., Ω_M) are still slightly larger than 10%. Ongoing and future SZ surveys will detect thousands of clusters (e.g., Birkinshaw 1999; Carlstrom et al. 2002; Bartlett et al. 2008), and this will significantly improve the constraints on cosmological parameters. Therefore, it is important to control the systematic uncertainties of galaxy cluster physics at even a percentage level.

Because of the very long Coulomb collisional timescale in the low-density outer regions of galaxy clusters, it has been pointed out that electrons and ions there can be in non-equipartition (Fox & Loeb 1997; Ettori & Fabian 1998). Numerical simulations have shown that the SZ observables, e.g., integrated Comptonization parameter (Y), for relaxed clusters can be biased by a few percent (Wong & Sarazin 2009), and can potentially be biased up to ∼10% in major merging clusters (Rudd & Nagai 2009). Specifically, the non-equipartition effect reduces the electron pressure compared to equipartition models, and hence the integrated Comptonization parameter of the non-equipartition model is smaller than that of the equipartition model. Although the uncertainties are still large, recent X-ray observations suggest that the electron pressure in cluster outer regions may be lower than that predicted by numerical simulations assuming equipartition (Basu et al. 2010; George et al. 2009; Hoshino et al. 2010). Recent observations of the secondary cosmic microwave background anisotropies with the South Pole

Telescope (SPT) and the *Wilkinson Microwave Anisotropy Probe* (*WMAP*) 7 year data also suggest that the electron pressure is smaller than the value predicted by hydrodynamic simulations (Lueker et al. 2009; Komatsu et al. 2010). These observational signatures are consistent with electrons and ions in non-equipartition, although it is also possible that the hydrodynamic simulations may simply overestimate the gas pressure. Another possibility is that heat conduction outside the clusters may be reducing the gas pressure (Loeb 2002).

In our previous paper, we have shown that the non-equipartition effect can introduce biases in the integrated SZ effect, and the biases depend on cluster mass and evolve with redshift in the ΛCDM cosmology. The non-equipartition model was discussed in detail in Wong & Sarazin (2009). In this paper, we study the impact of non-equipartition effect on precision cosmology studies using the non-equipartition models we have developed. We consider only the non-equipartition effects associated with an accretion shock at the outer edge of a cluster. No significant collisionless electron heating at the accretion shocks is assumed, and the intergalactic gas outside the collisionless accretion shock is taken to be cold. These assumptions maximize the non-equipartition effects of the accretion shock. On the other hand, cluster mergers are not considered in our work. Mergers may increase the non-equipartition effect by a few percent, and hence our calculations of the biases in cosmological parameter estimation may still underestimate these effects. However, the non-equipartition effect induced by mergers lasts for only 0.5–1 Gyr (Rudd & Nagai 2009), which is comparable to the timescale that mergers can temporarily enhance the integrated SZ effect by boosting the overall temperature (Wik et al. 2008, hereafter WSR). Such transient phenomenon will mainly introduce scatter in the Y versus mass relation, and the effect on cosmological parameter estimation is small in general. The non-equipartition effect induced by mergers may partially cancel out the merger boost in SZ effect, and hence the merger effect on cosmological parameter estimation may even be smaller. On the other hand, the non-equipartition effects in the accretion shock regions can systematically bias the Y versus mass relation for all clusters, as long as clusters are continuously accreting materials from the surrounding which is believed to be generally true. Moreover, systematic uncertainties in precision cosmology using galaxy clusters can be minimized by restricting the sample of clusters to the highest degree of dynamical relaxation, and hence considering the systematic effects on relaxed clusters alone is particularly important. We follow Randall et al. (2002, hereafter RSR) and WSR closely to quantify the biases in cosmological parameter estimation using semi-analytical techniques. Specifically, we study the non-equipartition effect on the Y versus mass relation (Section 2). Such a biased Y versus mass relation will affect the number of clusters with Y observed in SZ surveys, i.e., the Y function (Section 3). In this work, the Y function is calculated using the extended Press–Schechter theory (Press & Schechter 1974). We consider three cases which may potentially introduce biases in the Y versus mass relations if the non-equipartition effect is not properly taken into account during the cross-calibration or self-calibration processes when using the galaxy cluster abundance technique (Section 4.1). We quantify and discuss the impact on cosmological parameter estimation from SZ surveys by fitting the mass function with the biased Y versus mass relations (Section 4.2). Section 5 gives the discussion and conclusions. Throughout the paper, we assume the Hubble constant $H_0 = 71.9\, h_{71.9}$ km s^{-1} Mpc^{-1} with $h_{71.9} = 1$.

2. SZ VERSUS MASS CORRELATION

The SZ effect can be characterized as the Comptonization parameter, y, which is given by

$$y = \frac{k_B \sigma_T}{m_e c^2} \int n_e T_e dl \propto \int P_e dl, \tag{1}$$

where σ_T is the Thomson scattering cross section, n_e is the electron number density, T_e is the electron temperature, $P_e = n_e k_B T_e$ is the electron pressure, and l is the distance along the line of sight. The integrated Comptonization parameter, Y, is defined as the integral of the Comptonization parameter in Equation (1) over the area of the cluster on the sky

$$Y = d_A^2 \int y d\Omega = \int y dA, \tag{2}$$

where d_A is the angular diameter distance to the cluster, Ω is the solid angle of the cluster on the sky, and A is the projected surface area. In this paper, Y is integrated over the projected surface area of the cluster out to the shock radius. This quantity is useful for spatially unresolved clusters with SZ observations where the beam area covers the whole cluster.

It has been shown that the integrated Comptonization parameter displays a tight correlation with cluster mass (Reid & Spergel 2006). Such a tight correction is useful for precision cosmology, and hence a correct understanding of the integrated Comptonization parameter is important. A detailed discussion of the use of SZ surveys to study cosmology can be found in Carlstrom et al. (2002). In this paper, we assume the SZ effect versus mass relation for the equipartition model to be the same as the equilibrium Y–M relation used in WSR. Specifically, the equipartition SZ effect versus mass relation we assume is of the form

$$Y_{\rm eq} = N\, x^{\alpha}\, p[x]\, h_{71.9}^{-2}\, {\rm Mpc}^2, \tag{3}$$

where $x = M_{200}/(h_{71.9}^{-1} 10^{15} M_\odot)$, N is the normalization constant, α is the power-law index, and $p[x]$ is a 13 degree polynomial in x. Equation (3) is fitted to the numerical solutions for the equilibrium Y–M relation in WSR. The integrated SZ bias introduced by the non-equipartition effect, $Y_{\rm non-eq}/Y_{\rm eq}$ versus M at different redshifts, is taken from Wong & Sarazin (2009). The non-equipartition SZ effect versus mass relation we used in this paper is hence given by

$$Y_{\rm non-eq} = Y_{\rm eq,WSR} \times \left(\frac{Y_{\rm non-eq}}{Y_{\rm eq}} \right)_{\rm WS}, \tag{4}$$

where the subscripts "WSR" and "WS" here indicate that the terms are taken from different models in WSR and Wong & Sarazin (2009), respectively. In this work, since we are interested in the relative effects on SZ surveys and cosmological parameter estimation introduced by the non-equipartition effects instead of the precise Y–M relation which depends on details of numerical simulations, in principle, we can take any equilibrium Y–M relation from simulations and apply our non-equipartition bias to the equilibrium Y–M relation. The reason for using the equilibrium Y–M relation in WSR to model the non-equipartition Y–M relation instead of the Y–M relation in Wong & Sarazin (2009) is that the former relation takes into account the dependence of Y on gas fraction $f_{\rm gas}$, where $f_{\rm gas} \propto M^{1/3}$ for $M_{200} \gtrsim 10^{14}\, M_\odot$. The numerical solutions in Wong & Sarazin

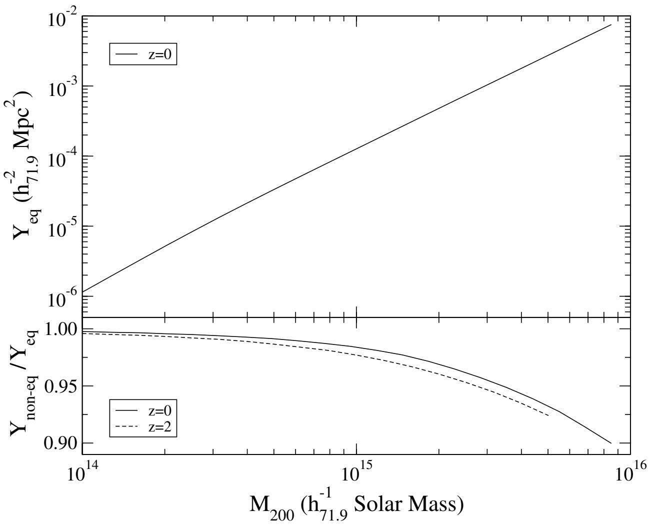

Figure 1. Top panel: integrated Comptonization parameter Y vs. mass at $z = 0$ for the equipartition model used in this paper. Bottom panel: integrated Y bias, $(Y_{\rm non-eq}/Y_{\rm eq})_{\rm WS}$, vs. mass at $z = 0$ and 2.

(2009) assume a constant $f_{\rm gas}$, and a self-similar argument shows that $Y \propto M^{5/3} f_{\rm gas} \propto M^{5/3}$. The equilibrium Y–M relation in WSR has a power-law index close to 2. On the other hand, the non-equipartition effects on the integrated Y depend weakly on $f_{\rm gas}$ and hence, we can assume the Y bias of the constant $f_{\rm gas}$ models in Wong & Sarazin (2009) can be applied to the varying $f_{\rm gas}$ model in WSR. Another advantage of using the equilibrium Y–M relation in WSR is that we can compare the effect of non-equipartition on cosmological parameters estimations to the merger effects calculated in WSR. The equipartition Y–M relation and the integrated SZ bias introduced by the non-equipartition effect we used in this paper are plotted in Figure 1. Clusters with higher masses are hotter, and hence, the equipartition timescales are longer. Thus, the non-equipartition effects are stronger in more massive clusters (Fox & Loeb 1997; Wong & Sarazin 2009). For our non-equipartition model in the ΛCDM universe, the integrated SZ bias decreases as redshift decreases. This is probably due to the decreasing rate of accretion onto clusters in the ΛCDM universe during the cosmological acceleration, which results in a longer time for electron–ion equilibration (Wong & Sarazin 2009).

In order to quantify the effect of non-equipartition on the SZ versus mass relation, we fit a power-law function to the integrated SZ bias of the form

$$(Y_{\rm non-eq}/Y_{\rm eq})_{\rm WS} = \Delta N \left(\frac{M_{200}}{10^{15}\, M_\odot} \right)^{\Delta\alpha} \tag{5}$$

to all the clusters with M_{200} between 10^{14} and $4 \times 10^{15}\, M_\odot$. The range is consistent with the cluster mass range used to study the mass function in X-ray observations (Vikhlinin et al. 2009). We also fit over the wider range of 10^{13} to $10^{16}\, M_\odot$ for comparison. The fitted coefficients ΔN and $\Delta\alpha$ correspond to the biases in the fitted SZ versus mass relation if the non-equipartition effect is not taken into account. The clusters in our sample are distributed roughly uniformly in the logarithm of the mass. The fitted results are listed in Table 1.

The effect of non-equipartition is to decrease Y for the high-mass clusters, and hence the effect on the SZ versus mass relation is to lower the power-law index by 0.01 to 0.016, which corresponds to a decrease of 0.5%–0.8% for the SZ

Table 1
Effects of Non-equipartition on the SZ–Mass Relation

	$(1–40) \times 10^{14}\, M_\odot$		$(0.1–100) \times 10^{14}\, M_\odot$	
z	ΔN	$\Delta\alpha$	ΔN	$\Delta\alpha$
0.0	0.977	−0.0126	0.967	−0.0117
0.5	0.971	−0.0164	0.963	−0.0122
1.0	0.971	−0.0152	0.964	−0.0109
2.0	0.970	−0.0158	0.969	−0.0085

versus mass relation with power-law index $\alpha = 2$. This is comparable to the 0.8% uncertainty of the power-law index derived from simulations combined with observations within M_{500} (Arnaud et al. 2009). The normalization is biased to compensate for the change in power-law index, and the bias is about 3% for the scaled mass of $10^{15}\, M_\odot$. For SZ surveys which measure Y out to the shock radius, e.g., observations with a spatial resolution poorer than typical shock radii, we have shown that the non-equipartition effect can introduce a small deviation in the measured Y–M relation. The deviations are small, but future SZ surveys with sufficient statistics may be able to detect such signatures. On the other hand, if clusters are spatially resolved and Y are measured within R_{200}, we have shown that non-equipartition effect is smaller than 1% for all clusters, and hence the bias in the Y–M relation is negligible (Wong & Sarazin 2009). A bias in measurement out to the shock radii but not within $\sim R_{200}$ will indicate that cluster outer regions may be in non-equipartition.

3. EFFECTS OF NON-EQUIPARTITION ON SZ SURVEYS

The number of galaxy clusters expected to be found per unit comoving volume depends sensitively on cosmology. The quantity which is convenient to describe the cluster number density is the mass function, $n(M, z)$, where $n(M, z)dM$ gives the number of clusters per unit comoving volume with masses in the range $M \to M + dM$, and z is the redshift. While the exact form of the mass function can be found most accurately from cosmological simulations (Springel et al. 2005), a semi-analytic form of the mass function given by the extended Press–Schechter theory (Press & Schechter 1974; Bond et al. 1991; Lacey & Cole 1993) provides a more convenient way to understand the dependence of the mass function on cosmological parameters, especially when we are interested in the relative effect instead of the precise values of the mass function itself. Although the Press–Schechter theory cannot reproduce the mass function found in cosmological simulations at very high redshifts and low cluster masses (Sheth & Tormen 1999; Lukić et al. 2007), it is more than sufficient over the redshifts ($z = 0$ to 2) and masses ($M = 10^{14}$ to $10^{16}\, M_\odot$) of interest here.

The mass function given by the extended Press–Schechter theory can be written as (Press & Schechter 1974)

$$n_{\rm PS}(M, z)dM = \sqrt{\frac{2}{\pi}} \frac{\bar{\rho}}{M} \frac{\delta_c(z)}{\sigma^2(M)} \left| \frac{d\sigma(M)}{dM} \right| \exp\left[-\frac{\delta_c^2(z)}{2\sigma^2(M)} \right] dM, \tag{6}$$

where $\bar{\rho}$ is the current mean of the total mass density of the universe, $\sigma(M)$ is the current rms density fluctuation within a sphere of mean mass M, and $\delta_c(z)$ is the critical linear overdensity required for a region to collapse at redshift z. Unless otherwise specified, the parameters used in this paper are the same as those in RSR and WSR.

Table 2
Characteristic Numbers of Clusters

z	z_l	z_u	$Y_0\left(h_{71.9}^{-2}\,\mathrm{Mpc}^2\right)$	$M_0\left(h_{71.9}^{-1}10^{15}\,M_\odot\right)$	$n_{\mathrm{PS},0}\left(h_{71.9}^{5}\mathrm{Mpc}^{-5}\right)$
0.0	0.00	0.25	6.10×10^{-4}	2.27	4.81×10^{-6}
0.5	0.25	0.75	5.95×10^{-4}	2.24	6.01×10^{-8}
1.0	0.75	1.25	1.97×10^{-4}	1.26	4.99×10^{-8}
2.0	1.25	2.75	6.04×10^{-5}	0.68	2.85×10^{-10}

Notes. The values of Y_0 and $n_{\mathrm{PS},0}$ are defined such that $\int_{Y_0}^{\infty}\int_{z_l}^{z_u} n_{\mathrm{PS}}(Y,z)\,dY\,(dV/dz)\,dz = 1$ and $n_{\mathrm{PS},0}(z) = n_{\mathrm{PS}}(Y_0,z)$ for the equipartition models for different redshift intervals z_l to z_u. The value of M_0 corresponding to Y_0 is related by Equation (3).

Once the Y versus mass relation is known, the distribution function of Y is given by the Y function,

$$n_{\mathrm{PS}}(Y,z) = n_{\mathrm{PS}}(M,z)\frac{dM}{dY}, \tag{7}$$

where $n_{\mathrm{PS}}(Y,z)dY$ gives the number of clusters per unit comoving volume at redshift z which have the integrated SZ parameters in the range $Y \to Y + dY$. The Y function for the non-equipartition models and the equipartition models can be related by

$$n_{\mathrm{PS,non-eq}}(Y_{\mathrm{non-eq}},z) = n_{\mathrm{PS,eq}}(Y_{\mathrm{eq}},z)\frac{dY_{\mathrm{eq}}}{dY_{\mathrm{non-eq}}}, \tag{8}$$

where the subscripts "eq" and "non-eq" denote the equipartition and the non-equipartition models, respectively.

Figure 2 shows the Y functions for the equipartition and the non-equipartition models and also their ratios at different redshifts for the standard ΛCDM cosmology. The theoretical Y functions can be biased strongly for large Y and high-redshift clusters. In practice, whether the bias can affect the observed Y functions depends on the number of clusters that can be observed, and this depends on cosmology. The maximum number of clusters that can be observed with Y values in the range $Y = Y \to Y + dY$ and with redshifts in the range $z = z \to z + dz$ is $n_{\mathrm{PS}}(Y,z)\,dY dV$, where dV is the comoving volume of the universe between redshifts z and $z + dz$. For each of the four redshifts $z = 0$, 0.5, 1.0, and 2.0, we selected a redshift interval z_l to z_u as given in the second and third columns of Table 2. Then, we defined values of Y_0 and $n_{\mathrm{PS},0}$ such that $\int_{Y_0}^{\infty}\int_{z_l}^{z_u} n_{\mathrm{PS}}(Y,z)\,dY\,(dV/dz)\,dz = 1$ and $n_{\mathrm{PS},0}(z) = n_{\mathrm{PS}}(Y_0,z)$ for the equipartition models. These values are listed in Table 2 and are plotted in Figure 2 as solid dots. For example, between $z = 0.25 \to 0.75$ for the assumed cosmology, the expected number of clusters with $Y \gtrsim 6 \times 10^{-4}\,h_{71.9}^{-2}$ Mpc^2 ($M \gtrsim 2 \times 10^{15}\,h_{71.9}^{-1}\,M_\odot$) is one. On the other hand, the expected number of clusters with $Y = (1 \to 2) \times 10^{-4}\,h_{71.9}^{-2}$ Mpc^2 [$M = (0.9 \to 1.3) \times 10^{15}\,h_{71.9}^{-1}\,M_\odot$] is about 200 within the same redshifts interval, and the bias in n_{PS} is about 5%.

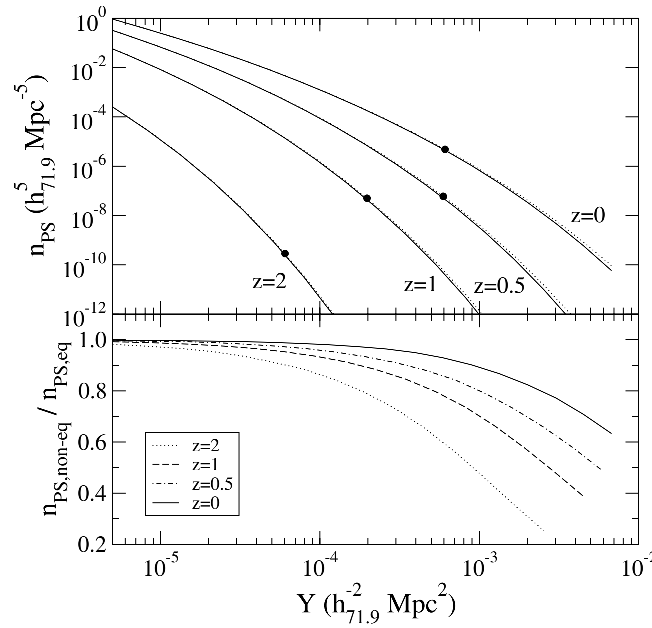

Figure 2. Top panel: integrated Y functions of the non-equipartition (solid lines) and the equipartition (dotted lines) models in the standard ΛCDM universe. Solid dots indicate the values of Y_0 such that $\int_{Y_0}^{\infty}\int_{z_l}^{z_u} n_{\mathrm{PS}}(Y,z)\,dY\,(dV/dz)\,dz = 1$ for the equipartition model. Bottom panel: ratios between the non-equipartition and equipartition Y functions.

4. EFFECTS OF NON-EQUIPARTITION ON COSMOLOGICAL PARAMETER ESTIMATION FROM SZ SURVEYS

In this section, we follow a procedure similar to that outlined in RSR and WSR to address the impact of non-equipartition effects on cosmological parameter estimation from SZ surveys. Readers who are interested in the technical details should refer to RSR and WSR. We outline the fitting procedure used in this work and address the difference between the previous works below.

As discussed in Section 3, the mass function of galaxy clusters is sensitive to cosmology, and hence measuring the galaxy cluster abundance at different redshifts can provide constraints to cosmological parameters. In particular, the properties of the dark energy can potentially be determined (Haiman et al. 2001). However, the masses of galaxy clusters cannot be directly determined, and a mass proxy must be observed to determine the mass through the mass–observable relation. Examples of mass proxies are the SZ temperature distortion, the X-ray flux, and the weak leasing shear. If the mass–observable relations can be calibrated, this can provide very tight constraints on cosmological parameters (Mantz et al. 2008; Vikhlinin et al. 2009). The mass–observable relations can be calibrated by numerical simulations and/or cross-calibrations with some other observables, and these are subjected to systematic uncertainties due to cluster physics and/or observational constraints. On the other hand, the mass–observable relations can be simultaneously fitted with the mass function of galaxy clusters and this is called "self-calibration" (Levine et al. 2002; Hu 2003; Majumdar & Mohr 2003). The sensitivity in constraining cosmological parameters and the mass–observable relations by "self-calibration" depends on both the form of the mass–observable relations and the mass function of galaxy clusters. In this work, we are mainly interested in the biases introduced by the non-equipartition effect, and the effects on the mass–observable relations. Hence, we assume there are no other systematic uncertainties in the mass function in any given cosmology.

4.1. Systematic Uncertainties Introduced by Non-equipartition

To address the impact of non-equipartition effects on cosmological parameter estimation from SZ surveys, we quantify the impact as biases in the cosmological parameter estimates if the calibration of the Y versus mass relation does not include the non-equipartition effect. We generate the integrated Y function with the non-equipartition effects included under an assumed cosmology, and call the generated Y function the non-equipartition Y function. We then fit the non-equipartition Y function with the incorrectly calibrated Y versus mass relation. We consider three different cases for the incorrectly calibrated Y versus mass relation. For the first case (case 1), we assume the Y versus mass relation is calibrated with incorrect numerical simulations that assume equipartition, and this incorrectly calibrated Y versus mass relation is used to fit the mass function. An example of this systematic bias might occur if the Y versus mass relation was extrapolated to the shock radius using observations within a smaller radius together with numerical simulations assuming equipartition. In this case, the integrated SZ bias is simply given by

$$b_1 = \left(\frac{Y_{\rm non-eq}}{Y_{\rm eq}}\right)_{\rm WS}, \tag{9}$$

where the right-hand side is the same term in Equation (4).

For the second case (case 2), we assume the Y versus mass relation is self-calibrated by fitting the Y versus mass relation and the mass function simultaneously, but with an incorrect functional form in the Y versus mass relation. We assume the incorrect functional form of the Y versus mass relation to be a power law in mass. In this case, we assume the integrated SZ bias is given by

$$b_2 = \left(\frac{Y_{\rm non-eq}}{Y_{\rm eq}}\right)_{\rm WS} \Big/ \left(\frac{Y_{\rm non-eq}}{Y_{\rm eq}}\right)_{\rm plfit}, \tag{10}$$

where the term with the subscript "plfit" is the best-fit power-law relation given in Equation (5).

For the third case (case 3), we assume the Y versus mass relation is calibrated correctly at $z = 0$, but the Y versus mass relation at higher redshifts is incorrectly calibrated by extrapolating the calibration from that at $z = 0$. In this case, we assume the integrated SZ bias is given by

$$b_3 = \left(\frac{Y_{\rm non-eq}}{Y_{\rm eq}}\right)_{{\rm WS},z} \Big/ \left(\frac{Y_{\rm non-eq}}{Y_{\rm eq}}\right)_{{\rm WS},z=0}. \tag{11}$$

Studying the impact on cosmological parameter estimation for all of these cases is equivalently to fitting the non-equipartition SZ luminosity function by the equipartition SZ luminosity function in Equation (8), but replacing the $dY_{\rm eq}/dY_{\rm non-eq}$ by b_1, b_2, and b_3 in Equations (9)–(11).

4.2. *Fitting Procedures and Results*

For each case of the systematic bias we studied, we generate the Y function with the non-equipartition bias included as given in Equation (8) by assuming the standard ΛCDM cosmological model with $\Omega_M = 0.258$, $\sigma_8 = 0.796$, and a constant dark energy equation of state parameter $w = -1$. The Y function generated can be directly calculated from the analytic Press–Schechter mass function in Equation (6) and the biased Y versus mass relations. This is simpler than those in RSR or WSR where merger trees were needed to generate the mass function in order to follow the merger history. In our work, all the Y versus mass relations are biased regardless of the merger history. We then fit the generated Y function with an equipartition model and find the best-fit values of the cosmological parameters. Clusters of galaxies can be used to constrain the dark energy equation of state parameter, w. Even the evolution of w can potentially be constrained. In this work, we study the constraint on $w(z)$ using the form $w = w_0 + w_1 z/(1+z)^2$; the detailed explanation of this choice can be found in WSR. We consider three cases when fitting the cosmological parameters: (1) fitting the Ω_M and σ_8 but fixing $w(z) = -1$; (2) fitting the Ω_M, σ_8, and assuming $w(z) = w_0$, where w_0 is a constant to be fitted; and (3) fitting the Ω_M, σ_8, and $w = w_0 + w_1 z/(1+z)^2$, where w_0 and w_1 are constants to be fitted. The Y functions are simultaneously fitted at four different redshifts ($z = 0$, 0.5, 1.0, and 2.0) to break the degeneracy in the fitted cosmological parameters. We choose only to fit Y between $5 \times 10^{-6}\, h_{71.9}^{-2}$ Mpc^2 and $5 \times 10^{-3}\, h_{71.9}^{-2}$ Mpc^2. The lower limit is chosen because clusters are likely to be confused in the SZ surveys for $M \lesssim 10^{14}\, h_{71.9}^{-1}\, M_\odot$ (Holder et al. 2007). Increasing the lower limit will make the biases in cosmological parameters larger since the non-equipartition effect increases with cluster mass. The upper limit corresponds roughly to the most massive cluster that can be formed in the ΛCDM universe. The limits are also consistent with SZ surveys being done or planned (e.g., Melin et al. 2005). We also limit the fits to values of $Y < Y_0$ and z such that the maximum number of observable clusters $\int_{Y_0}^{\infty} \int_{z_l}^{z_u} n_{\rm PS}(Y, z)\, dY\, (dV/dz)\, dz \gtrsim 1$ for most redshift bins (Table 2).

Non-equipartition Y functions at several redshifts and fitted models are shown in Figure 3. The deviations in the non-equipartition and the fitted Y functions are small and only visible in the residual plots. In cases 1 and 2 with the dark energy equation of state parameter frozen at $w = -1$, for $z \leqslant 1$, the non-equipartition Y functions are higher than the best-fitted Y functions for low-mass clusters, and the opposite is true for high-mass clusters. At $z = 2$, the non-equipartition Y function is slightly higher than the best-fitted one in case 1 with w frozen at $w = -1$, but the opposite is true in case 2. The residual is similar in appearance if we free the constant value of w (w_1 frozen at zero) or allow w to vary with redshift in cases 1 and 2. In case 3, the residual is more complicated. The residuals are of the order of a few percent, and this will affect the estimated cosmological parameters as discussed below.

The fitted results of the cosmological parameters for different cases we considered are summarized in Table 3. In general, for all three systematic uncertainty cases we studied, when freezing $w = -1$, the deviations of Ω_M and σ_8 from the assumed cosmology are $\lesssim$1%. The best-fitted parameters happen to be consistent with the assumed cosmology, but we can see from the residual plots that there are clear systematic deviations by a few percent (e.g., Figure 3). Such systematic deviations might be confused with the effect of non-Gaussian initial conditions on galaxy cluster mass functions, which also show similarly shaped systematic deviations (e.g., Figure 1 in Fedeli et al. 2009).

If we free the constant value of $w = w_0$, non-equipartition effects can be significant depending on how the calibration is done. For case 1, Ω_M, σ_8, and w_0 now deviate by +3.3%, −1.6%, and −5.4%, respectively. This shows that ignoring the non-equipartition effects in the Y versus mass relation when cross-calibrating with numerical simulations can introduce significant biases in cosmological parameter estimations when one is trying to constrain the dark energy equation of state. Either self-calibrating using a power-law form in the Y–M relation (case 2) or calibrating the Y–M relation correctly at low redshift (case 3) can significantly reduce the biases in Ω_M, σ_8, or w_0 (down to $\lesssim$1%).

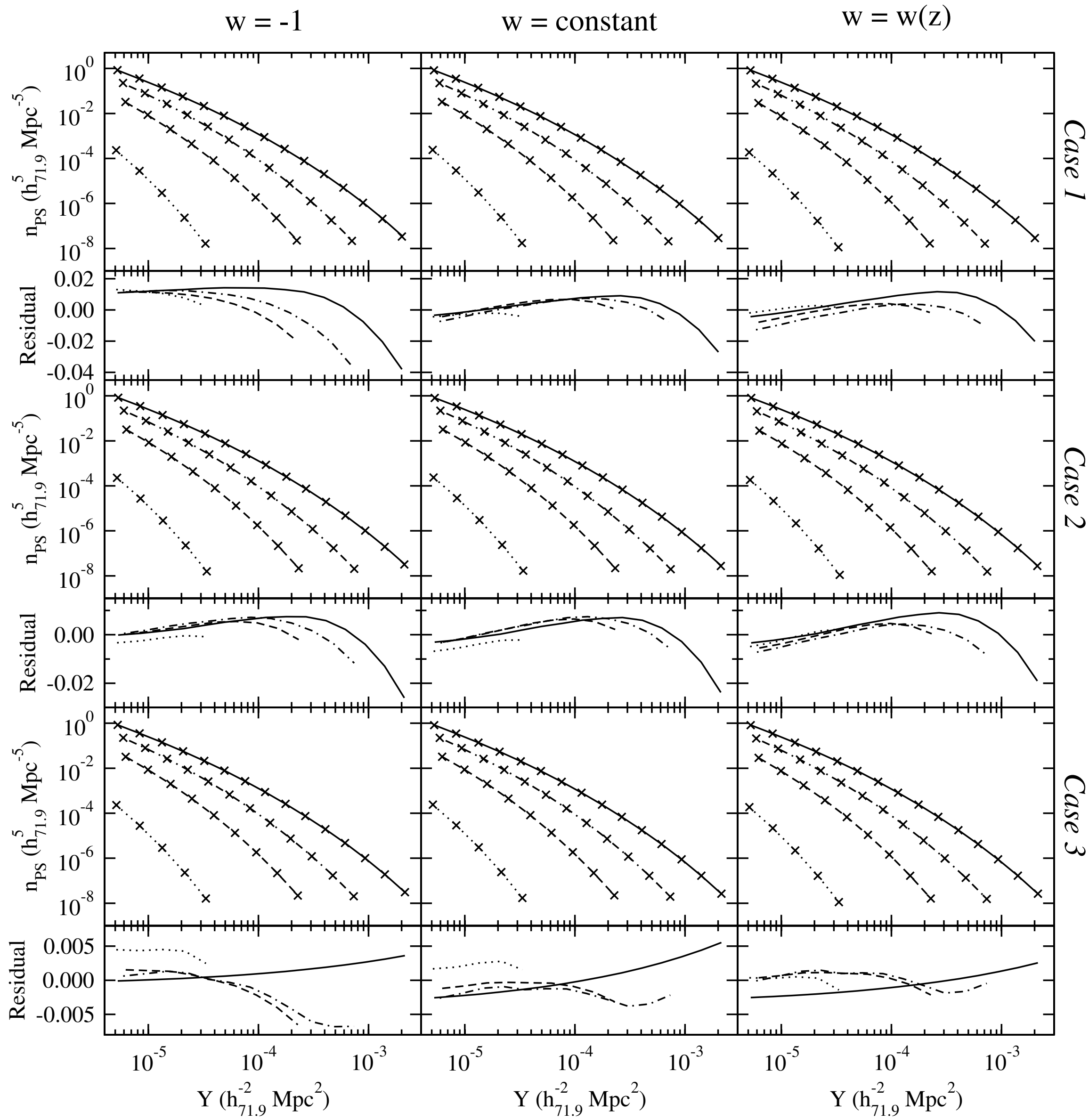

Figure 3. Non-equipartition Y functions (crosses) and the best-fitted Y functions (lines) at different redshifts are plotted on each figure in row 1, 3, and 5 for cases 1–3, respectively. Residuals in the log of the non-equipartition Y functions are plotted under the corresponding figures. Figures in Column 1 correspond to models with dark energy parameters frozen at $w = 1$. Figures in Column 2 correspond to models with w fitted as constant parameters (w_1 frozen at zero). Figures in Column 3 correspond to models with w allowed to vary with redshift. Redshifts at $z = 0$, 0.5, 1, and 2 are shown in solid, dash-dotted, dashed, and dotted lines, respectively.

Table 3
Best-fit Cosmological Parameters and Biases for Different Y Versus Mass Calibrations

Calibration	Ω_M	σ_8	w_0	w_1
Case 1	0.2548(−1.2%)	0.7950(−0.1%)	[−1]	[0]
	0.2665(+3.3%)	0.7830(−1.6%)	−0.9464(−5.4%)	[0]
	0.2680(+3.9%)	0.7811(−1.9%)	−0.9031(−9.7%)	−0.2362(+5.9%)
Case 2	0.2579(0%)	0.7976(+0.2%)	[−1]	[0]
	0.2602(+0.9%)	0.7951(−0.1%)	−0.9890(−1.1%)	[0]
	0.2610(+1.2%)	0.7940(−0.3%)	−0.9577(−4.2%)	−0.1725(+4.3%)
Case 3	0.2585(+0.2%)	0.7953(−0.1%)	[−1]	[0]
	0.2604(+0.9%)	0.7932(−0.4%)	−0.9910(−0.9%)	[0]
	0.2601(+0.2%)	0.7938(−0.3%)	−1.0123(+1.2%)	0.1167(−2.9%)

Notes. The assumed correct cosmological parameters are $\Omega_M = 0.258$, $\sigma_8 = 0.796$, $w_0 = -1$, and $w_1 = 0$. The bracketed values are the frozen values in the fits. The values in parentheses in Columns 2–4 are the percentage deviations of the fitted cosmological parameters from the assumed parameters. The values in parentheses in Column 5 are the largest percentage change in w between the present time ($z = 0$) and $z = 2$; this change is $\Delta w = w_1/4$ assuming $w(z) = w_0 + w_1 z/(1 + z)^2$.

If we allow w to vary with redshift, non-equipartition effects can introduce significant biases in cosmological parameter estimates (up to $\sim$10%). Self-calibrating using a power-law form in the Y–M relation (case 2) can reduce the biases in Ω_M and σ_8 down to $\lesssim$1%, but non-equipartition effects can still introduce a $\sim$4% bias in the constant normalization of the dark energy equation of state parameter (w_0) and introduce an apparent evolution of the same order. Calibrating the Y–M relation correctly at low redshift (case 3) can further reduce the bias in w_0 to $\lesssim$1%, but again, there is still an apparent evolution of $\sim$3%. These results show that the mass function technique is very sensitive to dark energy, and a full understanding of the systematic uncertainties in galaxy cluster physics is essential to constrain the dark energy equation of state using the mass function technique.

5. DISCUSSION AND CONCLUSIONS

Numerical simulations have shown that the SZ observables (e.g., the integrated Comptonization parameter Y) for relaxed clusters can be biased by a few percent (Wong & Sarazin 2009), and potentially up to $\sim$10% in major merging clusters (Rudd & Nagai 2009). These results are consistent with the SPT and the *WMAP* 7 year data which indicate that electron pressures are smaller than predicted by hydrodynamic simulations (Lueker et al. 2009; Komatsu et al. 2010). A few X-ray observations in the cluster outer regions also show that the electron pressure is lower than the pressure predicted by numerical simulations which assume equipartition (Basu et al. 2010; George et al. 2009; Hoshino et al. 2010). However, it is also possible that numerical simulations simply overestimate the gas pressure, or that the gas is supported in part by other forces such as turbulent or cosmic ray pressure. The Y–M relation can be biased if non-equipartition effects are not properly taken into account. Precision cosmological studies using the evolution of the galaxy cluster abundance rely on a full understanding of the mass–observable relations, if the mass of the galaxy clusters cannot be directly measured as is usually the case in practice. We have studied systematically the impact of biased Y–M relation introduced by the non-equipartition effect on SZ surveys and on precision cosmological studies.

While previous studies show that the Y–M relation is stable to complicated physical processes such as mergers and the power-law index in the Y–M relation is robust (Poole et al. 2007; WSR), we have shown that non-equipartition effect can introduce a deviation from the power law in the Y–M relation. We have fitted a power law to the non-equipartition Y–M relation, and we found that there is a $\sim$1% bias in the power-law index, which is comparable to the uncertainty of the power-law index derived from simulations combined with observations within M_{500} (Arnaud et al. 2009). Such a small systematic bias and deviation from the power-law form have important implications for SZ surveys and precision cosmological studies using the SZ surveys.

Using the analytic extended Press–Schechter theory to quantify the mass function of galaxy clusters, we have studied the non-equipartition effects on SZ surveys. We found that the Y functions can be biased strongly for large Y and high-redshift clusters. For example, the expected number of clusters with $Y = (1 \rightarrow 2) \times 10^{-4} h_{71.9}^{-2}$ Mpc^2 [$M = (0.9 \rightarrow 1.3) \times 10^{15}\, h_{71.9}^{-1}\, M_\odot$] between $z = 0.25 \rightarrow 0.75$ can be biased by $\sim$5%. The net effect is that ignoring non-equipartition effects underestimates the abundance of high mass and high-redshift clusters. Cosmological parameters measured by using the cluster counting technique from SZ surveys will be biased if non-equipartition effects are not taken into account.

We have quantified the impact of non-equipartition effects on cosmological parameter estimations from SZ surveys by the galaxy cluster abundance technique using biased Y–M relations. We considered three potential systematic biases in the Y–M relations if the non-equipartition effect is not properly taken into account during the cross-calibration or self-calibration when using the galaxy cluster abundance technique. The best-fit cosmological parameters, Ω_M, σ_8, and also the dark energy equation of state parameters [$w = w_0 + w_1 z/(1 + z)^2$] using the biased Y–M relations were determined. For all the three methods of calibrating the Y–mass relation we have studied, if the dark energy equation of state parameter is frozen at $w = -1$, we find that the best-fit Ω_M and σ_8 are consistent with the assumed cosmology to within $\sim$1%. However, there are clear systematic deviations of a few percent in the fitted Y functions which may be confused with others effects such as non-Gaussian initial conditions (Fedeli et al. 2009). Models with non-Gaussian initial conditions predict that the actual number of clusters with higher mass can be lower than the models with Gaussian initial conditions (e.g., Figure 1 in Fedeli et al. 2009); our non-equipartition model predicts an apparent smaller Y for high-mass clusters and this underestimates the number of high-mass clusters if the non-equipartition effect is not taken into account. Note that at such small levels of systematic deviations, other systematic uncertainties such as the use of different mass functions (e.g., Press & Schechter 1974; Sheth & Tormen 1999) may introduce larger systematic biases. However, Fedeli et al. (2009) also used the Press–Schechter mass function (as we do) to determine the effect of non-Gaussianity. Thus, we can directly compare these effects with the results of non-equipartition, free of biases introduced by the choice of mass function, and estimate the bias non-equipartition will introduce in efforts to detect non-Gaussian fluctuations with clusters.

If w is fitted as a constant parameter (w_1 frozen at zero), then depending on the calibration methods, non-equipartition effect can introduce a few percent biases on the measured cosmological parameters (case 1). Either self-calibrating the Y–M relation using a power-law form (case 2) or calibrating the Y–M relation correctly at low redshift (case 3) can significant reduce the biases in Ω_M, σ_8, or w_0 to $\lesssim$1%. If we allow w to vary with redshift, the non-equipartition effect can introduce a bias in cosmological parameter of up to $\sim$10% (case 1). In particular, non-equipartition effects can introduce an apparent evolution in w of a few percent in all of the cases we considered.

Using the cluster abundance technique alone, an X-ray survey with 85 X-ray clusters has already constrained some cosmological parameters down to 1% level in statistical uncertainty (Vikhlinin et al. 2009). Ongoing and future SZ surveys will detect thousands of clusters (e.g., Birkinshaw 1999; Carlstrom et al. 2002; Bartlett et al. 2008), and this will significantly improve the constraints on cosmological parameters. Therefore, it is important to control the systematic uncertainties of galaxy cluster physics at even a percentage level. Hydrodynamic simulations assuming equipartition suggest that the integrated Y is a robust mass proxy even when galaxy clusters are in the process of merging, and hence the integrated Y is taken to be a nearly ideal probe for cosmological studies (Poole et al. 2007; WSR).

Our results show that if the non-equipartition effect is not properly taken into account, cosmological parameters can be biased significantly (up to $\sim$10%). In order to take the non-equipartition effect into account when using cluster abundance to study precision cosmology, the ultimate solution is to include the non-equipartition effect in cosmological simulations assuming the non-equipartition physics is known accurately. If higher resolution is needed, another approach is to correct the non-equipartition effect by performing idealized simulations (e.g., Wong & Sarazin 2009) or to re-simulate representative clusters taken from cosmological simulations including the non-equipartition effect with realistic assumptions. For the latter case, the non-equipartition effect can be taken into account together with other physical processes (e.g., gas depletion processes during the formation) which may also affect the Y versus mass relation. However, the above calibration methods by numerical simulations rely on the assumption that the non-equipartition physics is known accurately, which is in fact not the case at present. One of the key systematic uncertainties is the electron heating efficiency at the collisionless shock, β. One way to constrain the non-equipartition physics is to make direct observations of accretion shocks, which is currently not feasible. We may constrain non-equipartition physics based on observations of other astrophysical shocks such as mergers shocks and supernova remnants. However, we have to assume these results apply to cluster accretion shocks, which may or may not be the case. Another route might be to perform plasma simulations (e.g., particle-in-cell simulations) to constrain the shock physics. However, to apply the plasma simulation results to cluster accretion shocks, a detailed knowledge of the pre-shock physics such as the magnetic field structure might be needed. Clearly, all of the above calculations are necessary to determine the range of the systematic uncertainties and the effects of the non-equipartition physics, and to constrain the form of the Y versus mass relation. These should be studied in the near future. Until numerical simulations can directly determine the effects of non-equipartition on the Y versus mass relation from first principles, we suggest either to self-calibrate the Y versus mass relation using a power-law form at each redshift bin (case 2), or to calibrate the Y versus mass relation correctly at low redshift (case 3). These will reduce the biases due to non-equipartition on Ω_M, σ_8, or w_0 to better than 1%. Important biases introduced by other physical processes can be corrected in addition to the non-equipartition correction. However, if one tried to constrain the evolution in w to better than 1%, together with the self-calibration method, constraints from numerical simulations with uncertainties less than a percent level might be necessary.

We have shown that using the cluster abundance to constraint the dark energy equation of state requires a full understanding of the systematic uncertainties in galaxy cluster physics. Even though we are only considering the systematic uncertainties introduced by the non-equipartition effect, our results also suggest that systematic uncertainties in the Y–M relation introduced by other physics of even a few percent can introduce a comparable level of biases in cosmological parameter measurements. Future cluster surveys aiming to constrain departures from general relativity will need to control systematic uncertainties down to a sub-percentage level (Schmidt et al. 2009), and hence cluster physics must be understood to a comparable accuracy. Future theoretical calculations and numerical simulations should pay particular attention to the effects of non-thermal physics on the electron pressure profiles. Potential systematic uncertainties include conduction, turbulent pressure, magnetic pressure, and relativistic pressure supported by cosmic rays. Deep observations should also be carried out to constrain all these effects in detail for individual clusters. The outer regions of galaxy clusters are ideal sites for study non-thermal physics. These studies not only can increase our understanding of cosmology but also can provide information on the physics of galaxy clusters and plasma physics under extreme conditions.

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