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A Quantitative Assessment and Comparison of the Undergraduate Curriculum Prerequisite Structures for the Universities in the Minnesota State System with Particular Emphasis on Mathematics Courses

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By

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Abstract

The purpose of this dissertation is to study the consistency of the structures and the centrality of mathematics courses in the curricula of the universities in the Minnesota State University system. This research will be based on the curriculum prerequisite networks for the seven universities in the Minnesota State System. These networks will be constructed from the information in the course catalogs available on each university's public website. The networks will be constructed with courses represented by nodes and weighted edges representing prerequisite relationships. The analysis will use curriculum network analytics to evaluate and compare the connectedness of the networks, the centrality of mathematics courses, and the importance of mathematics departments in the structure of each university's curriculum.

Acknowledgments

Of course, first and foremost, I have to thank my beautiful and understanding wife, Christina, for putting up with my pursuit of this degree over these last few years. Thanks also to Jason for keeping me in the program and dealing with my writing style. Opening a textbook as a student again was not an easy task, but I am glad that I did it. Partial responsibility also goes to my parents who gave me everything I ever needed. Unless I go crazy, this is my last degree. I hope this inspires my boys to do great things. Salutations and thanks also to Mrs. Lund who always challenged and believed in me.

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Chapter I - Introduction

Background of the Problem

It is typical for universities in the United States to have a list of regularly offered courses compiled into a course catalog. The course catalog will generally provide a description for each course along with any prerequisite or corequisite courses. Students must successfully complete a prerequisite course before starting work in the next course which will be called a target course. Some course prerequisites are satisfied with a passing grade, and some have a minimum grade requirement. Corequisite courses are similar to prerequisites, but often corequisites can be completed either before a target course or at the same time as the target course. Prerequisite courses can lead to a chain of courses that must be completed, one after another, before a student can proceed to the final course in the sequence. These prerequisite chains can lead to longer completion times for graduation. Each link in a prerequisite chain is likely to indicate another semester in a student's journey through the curriculum. Additionally, if a single course appears in several prerequisite chains, then it starts to take on a more central role in a university's curriculum. If a course that is central to the curriculum is problematic with regard to student success, then it can cause a bottleneck for the flow of students on their journey toward graduation. Alternatively, a course that is central to the curriculum can also be an important hub for the dissemination of knowledge throughout large parts of the university. The same ideas regarding a course being central to a curriculum can also be applied to departments that have courses that show up in several prerequisite chains. Since nearly every university now offers its course catalog in a digital format, a university's curriculum can be downloaded and modeled into a network where the nodes represent courses, and the directed edges represent the relationships between prerequisite and target courses. This prerequisite curriculum network can then be analyzed to give a quantitative representation for the structure of the university's curriculum. This analysis can then be used to help the university make more informed decisions on things ranging from curriculum changes to course scheduling and allocation of funding (Slim et al., 2014).

Prerequisites

Requirements for prerequisite knowledge can be found at universities throughout history. Often, prerequisite requirements were imposed upon students before enrollment in the university was allowed. For example, since Latin was the primary language of instruction at Medieval universities, students often had to satisfy a requirement of fluency in Latin before admission. One university admission requirement of the time stated that the Latin skills of prospective students must be developed enough to "read, sing and construe well and also compose twenty-four verses on one subject in one day" (Lukas, 2006, p. 48). The curricula of early American universities were largely copied from Europe. This resulted in continued admission requirements of fluency in Latin and possibly Greek. It was not until 1745 that a university in North America required more than a working knowledge of these languages as a minimum for admission. In that year, Yale became the first American university to require some knowledge of mathematics as an admissions requirement (Denham, 2002). As the American university progressed through the nineteenth century, the typical university curriculum evolved from a strictly defined path to graduation to an elective model of education (Bok, 2013). As academic majors became more prevalent and courses have become more specialized, the act of choosing, assigning, and enforcing course prerequisites has become more complex for each university.

The reasons for selecting and assigning modern course prerequisites vary. Many prerequisite courses that are contained in the same academic department as their target course will be chosen due to a direct link in content between the two courses. This is often seen in sequential courses with titles that can be similar to Elementary French I and Elementary French II (Abou-Sayf, 2008; Walker, 2010). The content learned in the prerequisite courses will be necessary for the students to understand the content in the target courses. Beyond these typical course content prerequisites, courses can also be selected as prerequisites for other reasons. A course may be chosen as a prerequisite because its completion has been shown to produce more successful or capable students in target courses. This can happen even though there is little to no content overlap between the prerequisite and target courses (Walker, 2010; Prante, 2016; O'Shea & Pollatsek, 1997). Walker calls these courses filter and maturity prerequisites. Additionally, a course may be chosen as a prerequisite to satisfy a safety requirement in a target course or to satisfy a requirement called upon by a stakeholder outside of the university (Abou-Sayf, 2008). Requirements from outside of the university may come from a system that the university belongs to (Minnesota State, 2015), a state legislature (Omnibus Supplemental Appropriations Bill, 2014), an outside accrediting body, or some other stakeholder with sway over desired student outcomes (Johnson & Wang, 2015).

Mathematics Curriculum

In science, technology, engineering, and mathematics (STEM) disciplines, mathematics courses are often called upon to act as prerequisites for major's courses. The

2015 Curriculum Guide to Majors in the Mathematical Sciences (Mathematical Association of America, 2015) noted that biology students accounted for 30% of mainstream Calculus I students with engineering students accounting for 27% of those same students. College Algebra is often one of the lowest level mathematics courses available for college credit at a university. It also serves as a prerequisite for classes in many departments. A study of one university by Herriott and Dunbar (2009) found that only 9% of students enrolled in College Algebra had declared majors in actuarial science, chemistry, computer science, engineering, mathematics, or physics. Over 70% of the students enrolled in College Algebra in the study by Herriott and Dunbar had declared majors in either Business, Economics, or Health Sciences. This research indicates that mathematics courses and departments play a central role in the structure of a university's curriculum and can greatly affect a student's progress towards graduation.

University mathematics departments tend to have courses that follow a relatively standard hierarchy of core courses that form a prerequisite chain through the mathematics curriculum. College algebra is often one of the lowest level mathematics courses that offers college credit. From college algebra, students can progress through the mathematics curriculum by taking Precalculus or Trigonometry, Calculus I, Calculus II, Calculus III, Differential Equations, and then Linear Algebra (O'Shea & Pollatsek, 1997). This typical path through the mathematics curriculum makes up part of what Goldberg (2008) called the math-science death march. Many STEM majors will start on this prerequisite chain at or after Calculus I, but that still leaves a long path to completion (Mathematical Association of America, 2015).

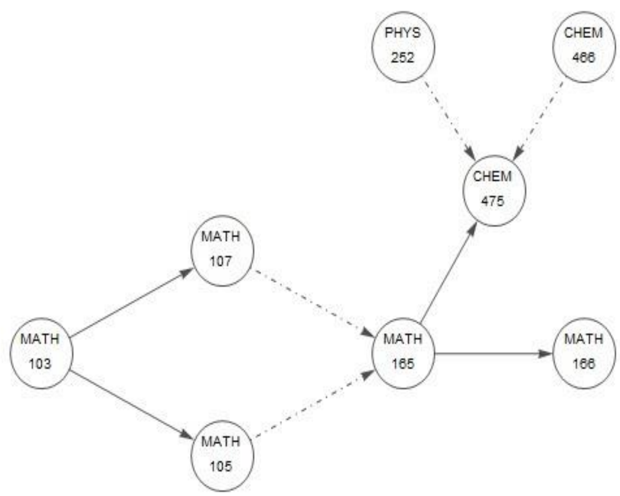
Network Analysis

With all of the course and prerequisite information for universities now available in online course catalog repositories, it is now possible to compile and analyze this information using methods that would have been difficult or impossible in the past. However, Knorn et al. (2019) indicate that quantitative measures of curriculum design are rarely used at the university level. Ruitenberg (2005) states that "cartographic representations are not common in educational theory, nor have the functions and effects of cartographic representation been fully considered and studied in educational circles" (p. 8). The cartographic representations that Ruitenberg refers to in this quote include networks and network analysis. Ruitenberg states that networks help represent the spatial aspects of education in ways that the written word cannot as easily convey. Aldrich (2015) comments that the use of concept and curriculum mapping has come close to representing a scientific view on the shape of curricula. However, Aldrich indicates that the visual representations of the structure represented in concept and curriculum mapping is too often hidden in text, tables, and spreadsheets. Further, concept and curriculum mappings do not easily lend themselves to quantitative analysis. In the last decade, the use of network analysis to describe university curricula has become more common. Some studies focus on the connections of concepts in learning but not the prerequisite connections from university course catalogs (Knorn et. al, 2019; Komenda et. al, 2015; Varagnolo et. al, 2020). Other studies use student enrollment and performance data along with networks built based on course prerequisite information to model things like student flow through a program or college (Saltzman & Roeder, 2011; Molontay et al., 2020). A third type of study uses metrics and clustering algorithms on curriculum prerequisite networks to describe the structure, connectedness, strengths, and issues with curricula (Aldrich, 2015; Heileman et al., 2019; Meghanathan, 2017; Slim et al., 2014; Wigdahl et al., 2014).

Representing the course catalog of a university with a prerequisite network provides unique visual representations and quantitative analyses of the curriculum (Aldrich, 2015). A network is made up of nodes (vertices) and the edges that form the connections between these nodes. The edges that connect nodes can be directed or undirected. A directed edge indicates a connection starting at one node and only moving to the other. An undirected edge indicates a two-way connection between two nodes. Edges and nodes can also be weighted. Different weightings can be assigned to the edges connecting two nodes (Newman, 2010).

Figure 1

Sample Portion of a Prerequisite Network and Its Adjacency Matrix



	MATH 103	MATH 105	MATH 107	MATH 165	MATH 166	CHEM 466	CHEM 475	PHYS 252
MATH 103	0	0	0	0	0	0	0	0
MATH 105	1	0	0	0	0	0	0	0
MATH 107	1	0	0	0	0	0	0	0
MATH 165	0	0.5	0.5	0	0	0	0	0
MATH 166	0	0	0	1	0	0	0	0
CHEM 466	0	0	0	0	0	0	0	0
CHEM 475	0	0	0	1	0	0.5	0	0.5
PHYS 252	0	0	0	0	0	0	0	0

Note. Image by the author.

Figure 1 is a sample portion for one possible setup of a directed prerequisite network. This prerequisite network shows the nodes as courses represented by their department and course number. Each directed edge starts at a prerequisite course and points to the target course. Solid lines indicate a required prerequisite course. Dashed lines indicate that the prerequisite can be satisfied from a choice of courses. For example, the prerequisite for MATH 165 is either MATH 107 or MATH 105 but both courses do not have to be completed. The prerequisites for CHEM 475 require MATH 165 and either PHYS 252 or CHEM 466. A student is not required to take both CHEM 466 and PHYS 252. Each of MATH 105, MATH 107, and MATH 166 have one required prerequisite course.

In Figure 1 each node represents a course, and each connecting edge represents a prerequisite relationship between the courses. These edges are inherently directed; the edge starts at the prerequisite course and points to the target course. The weighting of the edges can be determined by the statement of the prerequisite. If a course is required without choice, then the prerequisite edge will have a weighting of one. If an option is given to satisfy a prerequisite by completing one of n possible course choices, then the weight of the directed edges from the options to the target will each have a weighting of $1/n$ (Aldrich, 2015). A network can also be represented as a matrix (see Figure 1). If a network has n nodes, then the matrix would be built with *n columns* and n rows where each row and column represent a single node. In an adjacency matrix, the weighting of the edge connecting the j node to the i node is put in the i,j entry of the matrix. If no connection exists, then the entry in the adjacency matrix is zero. The analysis of the network is largely done using computer algorithms acting on matrices that represent the network (Newman, 2010). The theory and computations behind the calculations in network analysis are well defined and reproducible with any number of computer algebra systems.

Problem Statement

Since each university typically uses a local process to assign prerequisite courses, it is possible that there are inconsistencies in prerequisites across a state system of universities. Additionally, the various stages of reviewing, adding, removing, and enforcing prerequisites lead to the possibility of different curriculum structures across a statewide system of universities. A review and comparison of the course prerequisite networks for universities in a given system could reveal consistencies and inconsistencies across the curricula in the system. Further, an analysis of these networks can lead to useful information regarding course, department, and university curricular needs (Slim et al., 2014). In particular, is there consistency in the way that mathematics departments fit into university curricula and the corresponding prerequisite course networks for a state-wide university system?

Hypotheses

This research will analyze the course prerequisite networks for the seven universities in the Minnesota State System. The analysis will test for commonalities and differences in the ways that mathematics departments fit into each of the networks.

The flow of prerequisites through a mathematics department is often relatively standardized across universities (O’Shea & Pollatsek, 1997). However, courses outside of the mathematics department can be assigned mathematics course prerequisites for varied reasons (Abou-Sayf, 2008; Walker, 2010; Prante, 2016). The 2015 Curriculum Guide to

Majors in the Mathematical Sciences (Mathematical Association of America, 2015) noted that biology, engineering, and economics majors accounted for over half of the students who take university Calculus 1 courses. Overwhelmingly, these students took Calculus 1 to satisfy a program requirement or course prerequisite. Similarly, one university reported that 98% of students enrolled in College Algebra take the class to satisfy a program requirement or course prerequisite (Herriott & Dunbar, 2009). This large amount of assigning lower-level mathematics courses as prerequisites could lead to high variability in course prerequisites between universities. Alternatively, Walker (2010) implies that higher level mathematics courses are often assigned as prerequisites to ensure that students have developed mathematical maturity as opposed to specific skills. This dichotomy leads to the first hypothesis of this research.

Hypothesis 1: It is hypothesized that, in mathematics departments, lower-level courses will exhibit a higher variance in being labeled a prerequisite than higher level mathematics courses.

Many students enroll in mathematics courses to satisfy program requirements or course prerequisites for majors outside of the mathematics department (Herriott & Dunbar, 2009; Mathematical Association of America, 2015). This can lead to longer paths through the curriculum and extended timetables for the completion of a degree. In a study by Slim et al. (2014), it was found that the ten highest ranking courses on a cruciality metric at a large university were all in the mathematics department. The cruciality metric was based in part on the longest path that contains a course. This leads to hypothesis two.

Hypothesis 2: It is hypothesized that the longest prerequisite paths through the curriculum will contain mathematics prerequisites.

The internal prerequisites of a mathematics department tend to follow a fairly standard pattern running through the courses of College Algebra, Precalculus, a three-course Calculus sequence, and finally higher-level courses (O'Shea & Pollatsek, 1997). Also, mathematics courses are often taken by students in majors outside of the mathematics department to fulfill course requirements (Mathematical Association of America, 2015). The number of times that the mathematics department has courses listed as prerequisites increases the importance of the department to the overall structure of the curriculum prerequisite network. This leads to the third hypothesis of this research.

Hypothesis 3: It is hypothesized that mathematics departments will have consistently high centrality in the network analysis.

The final hypothesis comes from the idea of specialized and service mathematics courses. It is not unusual for mathematics departments to have courses specifically designed for students in another department or major. These service courses can serve as an alternative to the typical College Algebra or Calculus 1 course prerequisites (Ackerman, Fenton, & Raymond, 2020). Also, there are cases where these specialized courses are not taught in the mathematics department but in the department of the student's major course of study (Klingbeil & Bourne, 2014; Albers, 2018). More specialized courses would seem to be likely at larger institutions with more options for staffing and scheduling. Therefore, we have the fourth hypothesis of this research.

Hypothesis 4: It is hypothesized that the interconnectedness of undergraduate curricula will be inversely correlated with institutional size.

Significance of Research

Consistently defined curriculum prerequisite networks can give an effective way for comparing the curricula of universities (Heileman et al., 2018). Heileman et al. further indicate that faculty can review these comparisons “leading to data-informed decision-making around curriculum reform” (p. 3). Further, a comparison of the curriculum prerequisite networks can help target specific issues that each university closely tracks. Molontay et al. (2020) refer to curriculum prerequisite networks by noting that “the structure of the network has a huge impact on dropout rates and on graduation times” (p. 491).

Limitations

This research is restricted to the curriculum and prerequisite courses defined in the online course catalogs for the seven universities in the Minnesota State University System. The information in the online course catalogs for these universities will be scraped in the fall of 2021. Prerequisites that indicate a need for instructor, department, or program approval are not included. Prerequisites that require a certain number of credits must first be completed in a department, program, or college are not included. Prerequisites referring to “previous experience with…” are not included. One-way corequisites are treated the same as prerequisites. Two-way corequisites (co-corequisites) are treated as a single course.

Although historical success rates of students in particular courses and with particular instructors can be useful in determining how crucial a course is to a university’s curriculum (Slim et al., 2014), those considerations and statistics are not part of this research.

Definitions

Acyclic Directed Network

A directed graph is one where each edge connecting two notes only crosses in one direction. An acyclic graph has no cycles. There is no path in an acyclic directed graph where a node can be both the starting and ending point (Newman, 2010).

Betweenness Edge Centrality

The betweenness edge centrality of an edge in a network is a measure of how many shortest paths between nodes contain the given edge (Newman, 2010).

Bipartite Network

A bipartite network consists of two different types of nodes. The edges in a bipartite network can only connect nodes that are of a different type. Edges can be directed or undirected. For example, if the nodes in a directed bipartite network represent either learning outcomes or courses, then the edges in the network must either connect a learning outcome to a course or a course to a learning outcome (Newman, 2010).

Blocking Factor

The blocking factor for a course in a prerequisite network is the number of unique courses, excluding itself, that exist in all possible paths starting at that course (Slim et al., 2014)

Centrality

The measure of a node’s importance in a network is its centrality. There are many different acceptable kinds of centrality measures in network analysis (Newman, 2010). This research will use in and out degree centrality as well as betweenness edge centrality.

Curriculum Prerequisite Network

A network where the nodes represent courses, and each directed edge represents a relationship from a prerequisite to a target course (Aldrich, 2015).

Community Detection (Clustering)

Community detection is the act of reducing a network into subgroups that contain many connections between nodes and few connections between groups (Newman, 2010). The community detection used in this research will be done through the FindCommunity command in the program *Mathematica.* The centrality model of this command uses betweenness edge centrality to define its communities (Jung, 2016; see also Fortunato, 2010).

Degree Centrality

The in-degree centrality of a node in a directed network is the number of edges that direct into the node. The out-degree centrality of a node in a directed network is the number of edges that begin at that node. The degree centrality of a node in an undirected network is the number of edges that touch the node (Newman, 2010).

Delay Factor

The delay factor for a node in a prerequisite network is defined as the length of the longest geodesic path that contains that node (Slim et al., 2014)

Geodesic Path

A geodesic path between two nodes is one that crosses the fewest edges while traveling from the starting node to the ending node. If the edges have weightings, then the geodesic path is the one with the smallest sum of edge weights (Newman, 2010).

Geodesic Edge Betweenness Centrality

The geodesic edge betweenness centrality (edge betweenness) of an edge in a network is the number of geodesic paths that contain that edge (Fortunato, 2010).

Isolated Course

An isolated course is one that has no prerequisites and does not serve as the prerequisite for any other course. These courses will be not connected to any edges in the curriculum prerequisite network and will have a total in-degree and out-degree of zero.

Linked Course

A linked course will either have at least one prerequisite or serve as a prerequisite to at least one other course. These courses will be connected to at least one edge in a curriculum prerequisite network and will have a total in and out-degree greater than zero.

Path Length

The path length referred to in this research will be the geodesic path length. The geodesic path between two nodes is the one that requires the fewest number of edges to be crossed. The length of the geodesic path is the number of edges crossed while traveling from the starting node to the last node (Newman, 2010).

Prerequisite

A course that must be completed before a student starts work in a target course. Generally, if one is allowed to enroll in a prerequisite and a target course during the same semester, the prerequisite is called a corequisite. For the purposes of this research, prerequisites and corequisites will be treated as the same thing. In the case of two-way corequisites, the two courses will be listed as a single course with a single node in the network analysis. For example: If BIOL 105 has a corequisite of the lab course BIOL 105L and vice versa, then the course will be listed as BIOL 105/BIOL 105L.

Target Course

A target course is one that has a course prerequisite. For example: If MATH 165 has a prerequisite of MATH 107, then MATH 107 is the prerequisite and MATH 107 is the target.

Weakly Connected Components

Two nodes are in the same weakly connected component of a network if there is a path connecting them.

Chapter II – Literature Review

In a university's curriculum, it is common to have one course that must be completed before a student is allowed to continue onto a second course. The required course is called a prerequisite and the second course will be referred to as a target course. Prerequisite courses in a university's curriculum can be assigned for several reasons. Prerequisite assignments are generally local university decisions. It is very common for departments to have sequential courses that serve as prerequisites for the courses that follow. Often in these departmental prerequisite assignments there is an amount of content overlap between a prerequisite and a target course. The prerequisite course will serve as an introduction to a topic that will be utilized or built upon in the target course. Outside of content overlap and preparation, prerequisites can also be assigned for other reasons. Some course prerequisites are chosen because success in the course has shown to lead to success in a target course or program. Similarly, a prerequisite could be assigned to a target course to require students to reach a certain level of academic maturity before continuing onto a target course.

If it happens that a single course is listed as a prerequisite for several different target courses, then that course tends to be more important to the structure of the university's curriculum. Similarly, if a course is part of several different prerequisite chains in the curriculum, then it will again become more important to the university's curriculum. These courses and their departments become more central to the curriculum as they get called more often as prerequisites. This centrality can make courses a valuable information hub to a large part of the university's curriculum. It could also lead to a bottleneck that slows a student's progress toward a degree.

University mathematics departments often follow a standard chain of prerequisites through the mathematics curriculum. Additionally, mathematics courses often serve as course prerequisites to many courses outside of the mathematics department. Mathematics courses are often assigned as prerequisites to courses in science, engineering, technology, economics, and finance. The most common courses listed as mathematics prerequisites are the lower-level courses of College Algebra and Calculus 1. Having mathematics prerequisites for courses outside of the department adds to the length of prerequisite chains for higher level courses and programs. This can add to the difficulty of graduating in a timely manner.

Prerequisites

At the university level, prerequisite courses can be assigned for target courses to serve a few purposes. One of the most common reasons for selecting a course prerequisite is content preparation (Abou-Sayf, 2008; Walker, 2010). *Content prerequisites* are especially common in sequential courses contained in a single department. In this situation, a portion of the content studied in a prerequisite course will be directly referenced or built upon in a target course. For example, it would make little sense to teach the language skills of an Elementary French II course if students don't have a sufficient background in the French language equivalent to the work in an Elementary French I course.

An additional reason for choosing a course prerequisite is a common belief that completing the prerequisite course will help assure students' success in a target course or program (Abou-Sayf, 2008; Walker, 2010; Prante, 2016). This reasoning is similar to the idea of content preparation, but in this form, there may not be a direct content overlap between a prerequisite and target course. In these cases, a prerequisite essentially serves as a registration gate to the target course. This gate could be in place for what Walker (2010) calls filtering or maturity purposes.

A *filter prerequisite* is used when completing a course generally equates to increased student success in a target course. However, there is not necessarily a direct content overlap between the prerequisite and target course. Prante (2016) states that a mathematics prerequisite can be used as “a filtering mechanism for enrolment in economics. That is, students with mathematical skills (even those not used in Principles of Economics courses) are more likely to succeed in economics simply because strong mathematics skills are correlated with skills that also predict success in economics” (p. 81).

A *maturity prerequisite* lies somewhere between a content and filter prerequisite. With a maturity prerequisite, there again may not be direct content overlap between a prerequisite and a target course, but the prerequisite may provide students “with solid technical and conceptual tools…” and “whet their appetite for further study” (O’Shea & Pollatsek, 1997, p. 566). Walker (2010) recounts his teaching of a computer science course that did not directly draw upon the course’s mathematics prerequisites, but “the mathematical sophistication of the students allowed substantial depth in describing and analyzing algorithms” (p. 15).

In addition to helping ensure student success, the completion of a *safety prerequisite* may be required for safety reasons in the target course (Abou-Sayf, 2008). For example, an aviation program would certainly need to have coursework explaining safety measures that are expected of pilots before student pilots would be allowed to go on solo flights. A final common occurrence for choosing a course or program prerequisite may be due to external requirements. An example of this can be seen in nursing curricula that have certification requirements of nursing students which are independent of the university where classes are taken. Any designated content, filter, maturity, or safety prerequisites could be removed if the assumptions that defined the assignment are shown to be false or for other administrative considerations.

The assignment of course prerequisites at a university is generally a local decision. When a course is developed or up for review, there is typically a department and/or committee approval process that must be followed on each campus. The choice of assigning or removing course prerequisites is often the job of the academic department that will teach the course (Bok, 2013). It is common to choose course prerequisites based on quantitative research. A common study for the effectiveness of a prerequisite will compare student success in a target course against the student's completion or omission of a possible course prerequisite (Donovan & Wheland, 2009; Green, Stone & Charles, 2009; McCarron & Burstein, 2017; McCoy & Pierce, 2004). Further research has been conducted that investigates the properties of a prerequisite that lead to student success. Shaffer et al. (2016) showed that students benefited in a target course only on those topics that were covered at length in a prerequisite course. Tai and Sadler (2007) found that the method of instruction in a prerequisite course affected the usefulness of prerequisite material in a target course.

There can be additional requirements on course prerequisites imposed by a campus, system, or state. For example, for many years, the California Community College system had a policy that mandated cross-departmental prerequisites could only be called in a target course if there was statistical validation of the prerequisite course's benefits in the target course. After more than a decade of use, the statistical validation requirement was relaxed in order to allow content review as the justification of a cross-departmental prerequisite. Content review continues to be the predominant method of choosing course prerequisites for colleges and universities across the country (Academic Senate for California Community Colleges, 2010).

In the Minnesota State System of Colleges and Universities, prerequisite course selection is largely determined by the individual colleges and universities. However, the state legislature has mandated some standardization of requirements in several four-year degrees (2014 Session Laws H.F. 3172). The goal of the legislation was to develop transfer pathways that would ease the transition for students from two to four-year institutions. This Transfer Pathways program led to a standardization in the first two years of coursework for students beginning selected majors at two-year institutions. Those students would then be able to seamlessly transfer to a four-year institution to complete the coursework for their major (Minnesota State, 2015).

There is not a great deal of research on systematic methods used to choose or justify prerequisites through content review. Abou-Sayf (2008) attempted to formalize the prerequisite content review process on his campus by developing a program where faculty chose learning outcomes instead of courses when reviewing prerequisite requirements. The program then listed courses that had overlap with the desired learning outcomes. Abou-Sayf was trying to remove any preconceived notions that faculty had about which courses should serve as prerequisites. The results of his study found that the overlap of concepts from prerequisite to target courses was minimal in the best of circumstances. In a review of the curriculum for a university engineering department, Johnson and Wang (2014) sought the input of faculty and industry professionals to review the department curriculum through the use of a design structure matrix. These stakeholders defined desired incoming and outgoing skills for courses in the department. This information defined the matrix structure which, in turn, helped identify potential changes and deletions to the current prerequisite structure of the curriculum.

Reasons for the removal of prerequisites can also vary. There are instances where course prerequisites can form unintended bottlenecks where students don't advance through the curriculum at the desired pace. If a barrier for the advancement of students through a program is a bigger negative than the positive that the prerequisite adds to student success, then a change in the prerequisite structure can be made. New or different prerequisites can be assigned or developed without a reduction of standards, quality, or graduates (Klingbeil & Bourne, 2014). Removing prerequisites can also result in increased enrollment and shorten paths to graduation (Abou-Sayf, 2008; Soria & Mumpower, 2012; Johnson & Wang, 2014). Although, McCoy and Pierce (2004) claim that the enforcement of prerequisite gates does not have a long-term effect on enrollment. Finally, a prerequisite should be removed if it is listed for multiple courses in a path to completion. A redundant prerequisite could cause problems with recency requirements and registration barriers (Aldrich, 2015).

Enforcement of Prerequisite Courses

Research has shown that newly enforced course prerequisites can lead to increased student success in target courses (McCoy & Pierce, 2004; Soria & Mumpower, 2012). Soria and Mumpower state that the enforcement of prerequisites varies widely by university campus. Campus prerequisite enforcement ranges from electronic gates on registration to requiring faculty members and/or advisors to police the registration of students into particular courses. Soria and Mumpower found that enforcing prerequisites improved student success in an introductory composition course. The prerequisite enforcement also led to more students completing a developmental course before enrolling in the introductory composition course. Soria and Mumpower indicate that this increase in developmental course work stems from more developmental advising opportunities. "While we acknowledge the prerequisite system does not guarantee developmental advising opportunities, we believe it serves as an important gateway for many students to benefit from developmental advising" (p. 37). McCoy and Pierce similarly found that biology success rates improved after prerequisites changed from latent to enforced.

Mathematics Curricula

The internal course prerequisites of a mathematics department tend to follow a fairly standard pattern running through the courses of College Algebra, Precalculus, a three-course calculus sequence, and finally on to higher-level courses. There are naturally other non-major courses in the mathematics department that fit into this progression, but these noted courses represent a relatively consistent prerequisites chain in American university mathematics departments (O'Shea & Pollatsek, 1997). O'Shea and Pollatsek contend that this traditional setup of mathematics curriculum has many shortcomings and could benefit from some adjustments. To date, it appears that the suggestions advocated by O'Shea and Pollatsek have not been adopted for most universities. The Mathematical Association of America (MAA), through its Committee on the Undergraduate Program in

Mathematics (CUPM), noted in the “2015 CUPM Curriculum Guide to Majors in the Mathematical Sciences” that most mathematics departments still follow the traditional “hierarchical nature of the mathematics curricula” (Mathematical Association of America, 2015, p. 55). The CUPM report also indicates that universities must “choose prerequisites that best fit their goals, student populations, and local resources” (p. 37).

The courses outside of the mathematics department which call mathematics courses as prerequisites may have more variation. Inside and outside of the mathematics department, prerequisite mathematics courses are often also satisfied by enrolling at the university with a sufficient score on the ACT, SAT, or some other standardized test (Soria & Mumpower, 2012; Minnesota State, 2018). Mathematics courses are often listed as prerequisites for courses in STEM (Science, Technology, Engineering, and Mathematics) departments and business-related departments (Donovan & Wheland, 2009; Green, Stone & Charles, 2007; McCarron & Burstein, 2017; McCoy & Pierce, 2004).

Deeken, Neumann, and Heinze (2019) found that among university STEM departments, there is a desire for incoming students to have at least a basic knowledge of mathematical content, processes, and the nature of mathematics. The mathematical content in this study varied from elementary fraction mastery to calculus, vectors, and matrices. The mathematical processes ranged from basic skills to mathematical proofs and problem solving. For the students’ view on the nature of mathematics the study found that the STEM faculty believed it was necessary that “students should possess a meta-knowledge (Level 1) about, for example, the necessity of mathematical precision with respect to definitions and argumentations and the central role of proving when generating mathematical evidence” (p. 35). Survey results from Fox and Roehrig (2015) noted that in physical chemistry “students struggle: because they lack the necessary mathematics background to make connections between the concepts and mathematics” (p. 1462). Non-mathematics departments will often use College Algebra, Calculus 1, or a developmental mathematics course as a prerequisite. The departments that are more computation heavy may call for an entire calculus sequence as a prerequisite, but a calculus sequence has been shown to drastically reduce the number of students that are available to progress through programs in a timely manner. The point of a calculus sequence is not always the calculus content and computation abilities. There is often a desire from faculty for their students to have a level of mathematical maturity (Fulkner, Earl, & Herman, 2019). However, it has been claimed that “the standard Calculus-Linear Algebra sequence was never designed to meet the needs of students who would not continue with mathematics courses” (O’Shea & Pollatesk, 1997, p.4). These mid-level mathematics prerequisites often receive complaints for being required, and then the content covered is never called upon in the target courses (Walker, 2010). This desire for mathematics maturity in target courses, the dislike of the disruption to the graduation cycle, and absence of content recall in part led Dudley (2018) to comment “is mathematics necessary? No, but it is sufficient” (p. 364).

Mathematics courses serving as prerequisites for introductory courses in other departments also ensures that mathematics is seen early on in an academic career. In their study regarding the importance of mathematics as a prerequisite to financial accounting, McCarron and Burnstien (2017) noted that without mathematics prerequisites students would tend to put off their mathematics requirements until they are very near graduation.

The delay of completing mathematics requirements until the final semesters of a graduation cycle leads to mathematics courses being more of a hurdle than a benefit.

College Algebra is one of the most common low level prerequisite courses taken by undergraduate students (Herriott & Dunbar, 2009). However, mathematics departments often offer alternatives for students who will not need the skills reviewed in a college algebra course. Some examples of these mathematics department service courses are Mathematics for Finance, Business Calculus, Mathematics for Poets, and Mathematics for Engineers (O'Shea & Pollatsek, 1997). The American Mathematical Society (American Mathematical Society, 1999) estimates that mathematics departments account for up to 7% of the instruction at a university. As a consequence, each semester between 25% to 45% of a university's students are enrolled in at least one mathematics course in any given semester. Thus, the assignment of mathematics courses as prerequisites in American university curricula greatly impacts student learning and progress towards graduation.

If the typical mathematics courses and these service courses prove to be too large of a hindrance for the advancement of students, some departments may take on teaching similar prerequisite courses on their own. The engineering department at Wright State University eliminated its first-year mathematics prerequisites and replaced it with a mathematics course taught in the engineering department. After the change, an increase in graduation rates, student motivation, and self-efficacy were reported. These advancements were seen over a range of demographics and ability levels (Klingbell & Bourne, 2014).

Network Analysis

The structure of the prerequisite requirements of a university's curriculum can be captured in various forms of network representations. These network representations can then be analyzed to study the underlying structure of a university's curriculum. A network representation of a university curriculum will consist of nodes and edges. The nodes will represent specific portions of the curriculum and the edges will represent connections between these curriculum pieces.

In a *bipartite curriculum* prerequisite network there are two types of nodes. Some nodes will represent courses and other nodes will represent learning outcomes or goals. The edges in a bipartite network indicate what learning outcomes need to be learned in courses and which learning outcomes need to be learned before starting a course. The visual representation of these networks shows the flow of the learning outcomes through the curriculum.

A second type of curriculum prerequisite network has all the nodes representing courses in the university curriculum and directed edges that indicate a prerequisite relationship between two courses. A directed edge starts at the prerequisites course and points to the target course. This type of curriculum prerequisite network can help see the extent to which a university's curriculum is connected. The analysis on this network can point to courses of heightened importance that can greatly affect a student's progress towards graduation.

In each type of network there are measures that single out nodes which are important to the structure and flow of the information. A node's centrality in the network can be measured in several different ways. Traditional methods of measuring a node's *centrality* include a node's degree and its betweenness. A node's *degree* is the number of edges that start or end at the node. A node's *betweenness* is the number of shortest paths between two other nodes that travel through the node. These measures can be built upon in a curriculum prerequisite network to more specifically find important nodes that represent courses. A course's *blocking factor* will be a measure of a course's tendency to prevent a student from taking other courses. A course's *delay factor* will be the tendency of student's failure in a course to require a delay in graduation. The *cruciality* of a course will be a measure that combines delay and blocking factors, and the *complexity* of a program will be the sum of the cruciality of its courses.

The use of network analysis to describe university curricula has become more common in recent years. Some studies focus on the connections of concepts in learning and not the prerequisite connections from university course catalogs (Knorn et. al, 2019; Komenda et. al, 2015; Varagnolo et. al, 2020). Other studies use student enrollment and performance data along with networks built based on course prerequisite information to model things like student flow through a program or college (Saltzman & Roeder, 2011; Molontay et al. 2020). A third type of study uses metrics and clustering algorithms on course prerequisite networks to describe the topology, strengths, and issues with curricula (Aldrich, 2015; Heileman et al., 2019; Lightfoot, 2010; Meghanathan, 2017; Slim et al., 2014; Wigdahl et al., 2014).

Knorn et al. (2019) used directed bipartite networks to develop a quantitative tool to aid in analyzing the learning flow in university curricula. In the study by Korn et al., the nodes represented either university courses or key learning content. Due to the makeup of the nodes, the networks in this study were referred to as *directed courses-*

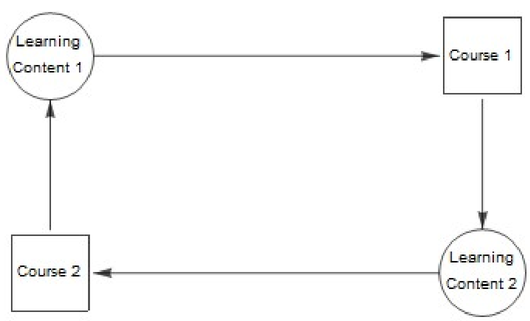
concepts graphs (DCCG). The directed edges in the DCCG then either went from learning content to a course or vice versa. A directed edge from learning content to a course represented prerequisite knowledge for that course and directed edges from a course to learning content indicated material taught in the course. Further, Knorn et al. used weighted edges to indicate the level of relevance of the learning content in the given course. A weighting of zero, one, or two indicated the degree that learning content varied from not relevant to very relevant in a given course. For prerequisite learning content this was a measure of how much the content would be relied on in the course. For learning content taught in a course, the weighting indicated how thoroughly the content was covered in the course.

Knorn et al. (2019) performed their analysis on a DCCG that represented seven courses and 111 learning concepts from the engineering curriculum at Uppsala University in Sweden. The analysis consisted of a search for cycles in the curriculum and interpreting the flow between nodes. Cycles found in the DCCG indicated that prerequisite learning content was being taught in a course that itself had prerequisite learning content being taught in the initial course (see Figure 2).

The flow of the DCCG is a reference of how well the directed "in" edge weightings match up with the directed "out" edge weightings for learning content (Newman, 2010). The assumption is made that the flow is optimized when the in and out edge weightings are equal in a path originating at a course, traveling through learning content, and terminating at a second course. This optimization of flow indicates that the focus on the content in the prerequisites class is reasonable for the focus that the content is given in the target course. If the in-weight is less than the out-weight on one of these paths, then the prerequisite course may need to add more focus on the content due to the importance of that content in the target course. If the in-weight is more than the out-weight, then the prerequisite course may be able to devote less time to that learning content due to its lesser importance in the target course (Knorn et al., 2019). Finding cycles and analyzing maximum flow in a DCCG can be done algorithmically by network analysis software once the nodes, edges, and weightings are defined. Korn et al. surveyed faculty to determine the weightings for the connections between courses and learning content.

Figure 2

A Cycle in a DCCG



Note. DCCG=Directed Courses Concepts Graphs.

Komenda et al. (2015) focused on course attributes, learning units, and learning outcomes in their research in an attempt to model and quantify the different structures and content of medical education programs. The study collected specifically designed and written text files regarding different learning outcomes and where they appear in the medical curriculum from different institutions. Data and text mining techniques were then used to find similarities between the documents describing different disciplines in the medical curriculum. The network representing this data had nodes that represented the different disciplines in the curriculum and undirected edges that represented similarities in course attributes, learning units, and learning outcomes, The edge weightings were larger for stronger similarities between disciplines. The network analysis involved using an algorithm to detect communities of disciplines in the curriculum and using centrality measures to define the vital parts of the curriculum.

The research by Komenda et al. (2015) uses the WalkTrap algorithm to define the communities in their developed network. The WalkTrap algorithm defines communities of nodes in a network based on what happens in randomized fixed length "walks" through the network. A walk through the network starts at a node and each step along a defined edge ends at another node. The WalkTrap algorithm defines communities in a network based on the idea that short random "walks" in the network will tend to start and stop in the same community (Latapy & Pons, 2006). Komenda et al. indicates that the communities in their network "represent the most crucial and important parts of a curriculum" (p. 5). Further, Komenda et al. states that in the visual representation of their network the "groupings of individual communities with related contents across the curriculum makes viewing the medical curriculum simpler and easier to understand" (p.14).

The centrality of a node is a measure of its importance in the structure of the network (Newman, 2010). There are several different ways to measure the centrality of a node in a network. Komenda et al. (2015) measure the centrality of the nodes in their network using closeness, betweenness, and eigenvector centrality. Newman indicates that the closeness centrality of a node in a connected network is defined as the reciprocal of the average of the geodesic distances from the given node to every other node in the network. A geodesic path between two nodes is one that crosses the fewest edges while traveling from the starting node to the ending node. If the edges have weightings, then the geodesic path is the one with the smallest sum of edge weights.

To adjust for the problem of a disconnected network, Komenda et al. (2015) define the geodesic distance between nodes that are not connected to be equal to the number of nodes in the network. Komenda et al. uses a low value of closeness centrality to identify those disciplines that are largely independent of others. Newman (2010) defines the betweenness centrality of a network node as the number of geodesic paths between any two nodes in the network that travel through the given node. Komenda et al. indicate that disciplines “with high betweenness centrality are best for joining the students’ knowledge from different collections of disciplines” (p. 8).

Newman (2010) states that *eigenvector centrality* is somewhat of an extension of degree centrality. The degree centrality of a node in an undirected network is the number of edges that connect to the node. In a directed network, those edges would be grouped into edges that start or end at the node to define in-degree and out-degree centrality measures. In eigenvector centrality, each node is given “a score proportional to the sum of the scores of its neighbors” (Newman, 2010, p. 169). The sum of these scores from connected nodes is the eigenvector centrality for the given node. The eigenvector centrality of a node can be high due to a node connecting to several other nodes or because a node connects to important notes. Important nodes would be ones that have high eigenvector centrality. It is because of this that Komenda et al. use eigenvector centrality to identify the important disciplines of the curriculum. Newman notes that eigenvector centrality works best for undirected networks.

Similar to the work of Knorn et al. (2019), the work of Varagnolo et al. (2020) organizes a bipartite network mapping of courses and concepts, but Varagnolo et al. investigates both the directed concepts-courses graphs (DCCG) and the undirected concepts-courses graphs (CCG). Again, the connections between courses and concepts were defined by faculty members with a score between zero and two with higher values indicating a more important concept in a course. This score serves as the edge weighting in the connection between concept and course nodes. The work by Varagnolo et al. on DCCG is similar to the work of Knorn et al. (2019) and indeed many of the authors are shared between the two papers. Much of the analysis of CCG by Varagnolo et al. focusses on centrality measures and a small amount of analysis is devoted to minimal cuts.

Varagnolo et al. (2020) reviewed degree, eigenvector, Pagerank, and betweenness centrality measures for the courses and concepts in the CCG developed for the Electrical, Computer, and Information Engineering program at Uppsala University in Sweden and the Engineering Physics program at Lulea University of Technology in Sweden. In their analysis, Varagnolo et al. noted that the degree centrality measure was not a desirable measure due to its sensitivity to the scoring from the faculty members. Varagnolo et al. states the scores that define the edge weights “should be chosen with great care and establishing a common understanding among teachers on how to assign weights is essential to retrieve meaningful insights from this specific metric” (p. 9). The closeness measure used by Varagnolo et al. is different from the definition used by Komenda et al.

(2015). Varagnolo et al. defines the closeness centrality of a node to be one over the sum of the lengths of the geodesic paths from the given node to all other nodes in the system. The difference in the two definitions is the numerator of the value being 1 or *n* (the total number of nodes in the network). In both cases, each node would have a closeness centrality measure between zero and one. Komenda et al. indicates that using the total number of nodes in the network as the numerator normalizes the measure for purposes of comparisons across different networks. Varagnolo et al. indicates that a high value of closeness centrality indicates a course node that shares common topics with many other courses or a concept node that is consistently taught and referenced across the curriculum. Varagnolo et al. also calculates the eigenvector centrality for the nodes in their networks. It is noted that in eigenvector centrality, scoring is increased more when a connection is to a higher scored node than when a connection is to a lower scored node. This measure of centrality will give a score to how influential a course or concept is to the curriculum.

PageRank centrality is named as such due to its use in the web ranking technology used by Google (Brin & Page, 1998). The idea of PageRank centrality has the scoring of a node increasing when there is a connection to a high scoring node, but nodes that connect out towards many other nodes do not increase the scoring by much (Newman, 2010). For example, if a course node connects to a concept node that, in turn, connects out to every other course, then that concept node will not increase the centrality of the course node by much. The PageRank centrality measure is noted to have results similar to degree and eigenvector centrality in the research by Varagnolo et al., but the interpretation of the measure is not otherwise expanded upon in the research. Similarly, the betweenness centrality measure in the Varagnolo research is noted to be of little interest due to the bipartite nature of the networks used.

Varagnolo et al. (2020) defines a *minimum cut* for a network as the minimal set of nodes or edges or combination thereof that would disconnect the network. A network is connected if there is a path from each node to every other node, and a network is disconnected if this is not true (Newman, 2010). In the bipartite networks used in the research of Varagnolo et al. a minimal course node cut would indicate courses vital to connecting the different areas and specialties of the curriculum.

In the course centrality results of the Varagnolo et al. (2020) research it is noted that the researchers and the studied departments interpreted the raw data as more of an indication of the flaws in the scoring process than a representation of course importance. Normalizing the edge weighting so that the sum of all edges connected to a course node is one made the degree centrality measure obsolete but fine-tuned the eigenvector and closeness centrality measures. Researchers and programming boards agreed that the courses with high normalized eigenvector and closeness centrality scores indicated the courses that were vital to the programs. It was noted that concept nodes with low closeness and comparatively high degree centrality were concepts important to the program but only intensively studied in a few courses.

In research on student flow through a program, Saltzman and Roeder (2011) used the prerequisite network for the core courses in the College of Business at San Francisco State University to investigate completion rates and time to graduation. The network that was used in this research had the core courses as nodes and the directed edges showed a relationship from a prerequisite to a target course. Saltzman and Roeder modeled how students flowed through the prerequisite paths of the core curriculum based on course enrollment patterns and student pass rates. The number of core courses taken per semester and the success rate for students were estimated from a random stratified sample of seniors in the College of Business. Multiple simulations were run to see if there was a statistically significant change to graduation time or completion rates based on changes to class size, prerequisites, and student preparedness. The flow model also helped to confirm which courses act as the largest bottlenecks for student's progression through the curriculum.

The research of Lightfoot (2010) organizes the curriculum of a college of business administration into a directed network where nodes represent courses and directed edges represent prerequisite relationships. Lightfoot then uses several graph analytic metrics to indicate courses that are uniquely important to the curriculum. Lightfoot uses the measures of in-degree, out-degree, betweenness centrality, eigenvector centrality, and clustering coefficients in his analysis of the directed prerequisite network. The courses with large in-degree are indicated to represent courses where higher level learning should take place and final program assessments should be given. Courses with large out-degree are indicated to be ideal for baseline assessments and introductory materials. Courses with large betweenness centrality are indicated to be "a key link between program tracks of course clusters within the program" (p. 67). The courses with high eigenvector centrality are said to be ideal points where material should be reinforced before assessment.

Lightfoot (2010) also uses the *clustering coefficient* for nodes in the prerequisite network to define courses that can be useful agents of change in connecting courses.

Newman (2010) indicates that the clustering coefficient has a somewhat misleading name. The clustering coefficient is not connected to how groups or clusters of vertices are identified in a network. The local clustering coefficient is actually more similar to the measure of betweenness centrality. Fagiolo (2006) states that the clustering coefficient measures the tendency of a network to form circles of connected nodes. Lightfoot’s paper does not provide the calculation used for the clustering coefficient, but there are typically two options to define the local clustering coefficient in a directed network. Newman defines one option where the directed edges of the network are considered without direction. This produces an undirected network, and the clustering coefficient of a node is then defined as the number of connected neighbors of the node divided by the total number of possible connections between those neighbors. Fagiolo (2006) calculates the local clustering coefficient of a node as the number of directed triangles formed with a vertex divided by the possible number of directed triangles that could include that vertex. A directed triangle between three nodes has directed edges connecting each of the three nodes. The directions of those edges indicate different directed triangles. This means that a clockwise path through the three nodes would be considered different from a counterclockwise path or three edges that do not form a continuous path. In either calculation the local clustering coefficient is a measure on the connectedness of the neighbors for a particular node. It is this measure of connection that leads Lightfoot to believe that the local clustering coefficient is a good indicator of courses that can enact changes to neighbor courses. In Lightfoot’s paper, representations of the prerequisite network are rendered for each of the metrics used where the node sizes represent the relative sizes of the measure for each course. The visual representation of the data leads to a more complete understanding of the measures and the network relationships and an easier identification of where the important courses in the curriculum lie. For the graph representing the clustering coefficient, it is noted that the course with the largest value would be a poor choice for enacting change. That course lands at the end of a prerequisite chain. The second largest clustering coefficient option is clearly pictured in the middle of a prerequisite chain and noted as the best candidate for changing the program. This analysis reiterates the idea that the measures of network analysis are often best interpreted when the overall structure of the graph is also considered.

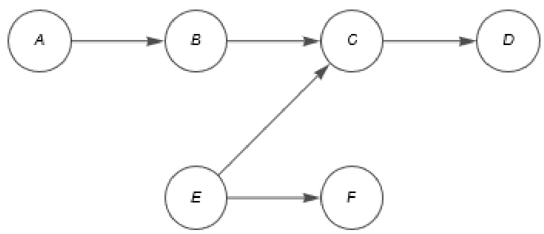
The research of Slim et al. (2014) introduces the metric of cruciality to university prerequisite networks. The prerequisite networks used by Slim et al. are initially built with courses being defined by nodes and directed edges representing prerequisite relationships. The cruciality metric of a course is defined by the sum of what Slim et al. call the course's blocking and delay factors.

A course's blocking factor is defined to be the number of the descendants a course has in the prerequisite network (Slim et al., 2014). If node *B* lies on a path after node *A* in a directed network, then *B* is defined to be a descendant of node *A*. In Figure 3, node *B* is a descendent of *A*. Similarly, the nodes *C* and *D* are also descendants of node *A*. Slim et al. include the blocking factor of a course in its cruciality measure due to the multiple paths towards graduation that are blocked if a high blocking course is failed. The delay factor for a course in a prerequisite network is defined by Slim et al. as the length of the longest geodesic prerequisite path that contains that course. This need not be a prerequisite path that starts with the course in question. In Figure 3, the delay factor for each of the nodes *A*, *B*, *C* and *D* is three due to the path from *A* to *D*. Slim et al. include the delay factor in the cruciality calculation due to the likelihood of a delay in graduating after a failure in a high delay course. If a two-year program contains a four-course prerequisite path as in Figure 3, then a student who is required to repeat any course in the path will have their completion of the program delayed.

Figure 3

Delay, Blocking, and Cruciality

Course	Delay Factor	Blocking Factor	Cruciality
A	3	3	6
B	2	3	5
C	1	3	4
D	0	3	3
E	3	2	5
F	0	1	1



Note. Adapted from Slim et al. (2014).

Slim et al. (2014) also includes a measure of *dissimilar mixing* based on the same directed network used in the cruciality measure. This is a measure calculated for a department, not for an individual course. The dissimilar mixing of an academic department is defined as the number of prerequisite edges that connect a department course to a course outside of the department divided by the total number of edges in the entire network. The connection from a department course to a course outside of the department can be in either direction. Slim et al. indicate that courses in departments with high dissimilar mixings will teach basic skills that should be mastered before students move on to more specialized courses.

Slim et al. (2014) also use a bipartite network to define the importance of a course. In these bipartite networks, the two types of nodes are courses and program curricula. Undirected edges indicate which courses are required in which programs. The importance of a course node is defined as the ratio of its degree to the number of program nodes in the network. Slim et al. includes this measure in the analysis to elevate those courses that may not be part of prerequisite paths but are required across several programs in the university's curriculum.

Heileman et al. (2019) used the cruciality measure from Slim et al (2014) to investigate the perceived quality of programs based on their complexity. Slim et al. defined the complexity of a program to be the sum of the cruciality of the courses in the program. Heileman et al. found that undergraduate electrical engineering programs with higher perceived quality had lower measures of complexity. In this study, the quality of a program was interpreted from the "U.S. News & World Report" ranking (U.S. News and World Report, 2021). Heilman et al. concludes that organizing a program with the least complex curriculum that still leads to students accomplishing the desired learning outcomes will yield the most student success. Heileman et al. notes that the relationship between low complexity and a high-quality program could also be a result of high-quality programs admitting high quality students that do not need the prerequisite work that a high complexity program requires.

Aldrich (2015) uses weighted directed networks to inspect the curriculum structure of Benedictine University in Lisle, Illinois with particular focus on the biochemistry and molecular biology programs. A network is weighted if different weightings are assigned to edges or nodes throughout the structure (Newman, 2010). In Aldrich's research, the nodes of the network represent courses and the directed edges point from a prerequisite to a target course. The weighting of the edges in this network is determined by the statement of the prerequisite. If a course is required without option as a prerequisite, then the edge representing that prerequisite is given a weight of one. If options are given to satisfy a course prerequisite, then each option is given an equal portion for the prerequisite edge weight. For example, if either Trigonometry or Precalculus will satisfy the prerequisite for Calculus 1, then each of the prerequisite edges representing those options would have a weighting of one-half. Aldrich used degree centrality, betweenness centrality, and connectedness to analyze the prerequisite network. Among other things, Aldrich found that about one-third of courses in the university's curriculum were grouped together in a weakly connected knowledge community. Nodes in a directed network are weakly connected if there are edges connecting them irrespective of the edge's direction. Outside of the largest knowledge community, there were several other smaller knowledge communities that were contained in single departments. Many of these small weakly connected communities contained a set of departmental courses that all called upon the same course as a prerequisite. This led to high degree centrality for these key departmental courses even though these courses were not connected to extra-departmental courses or the largest knowledge community in the network. Aldrich also noted that over half of the courses in the curriculum had no prerequisite connections at all.

Conclusion

In higher education, there are many situations where a network structure can help organize information, and network analysis can often help reveal some of the underlying structure of a topic. This research will use curriculum prerequisite networks where node's represent courses and the weighted directed edges represent prerequisite relationships. Research similar to this has been completed regarding particular universities and programs, but it does not appear that there has been much work done with respect to comparing all of the universities across a state system. This research will seek to compare the curriculum prerequisite networks for the seven universities across the Minnesota State System. Comparisons will be based on curriculum connectedness, cruciality, complexity, clustering, and centrality. Particular emphasis will be placed on mathematics courses and departments in the structure of the university's curriculum.

Chapter III - Method

This study will investigate the consistency of the placement and structure of mathematics prerequisites in the curriculum prerequisite networks at universities in the Minnesota State (MinnState) system. This study will use archival data scraped from each university's public website to build the course prerequisite networks for each university's curriculum. The quantitative and visual data will then be analyzed to test the following hypotheses. First, it is hypothesized that lower-level courses in mathematics departments will exhibit higher variance in being labeled a prerequisite than higher-level courses. Second, it is hypothesized that the longest prerequisite paths through the curriculum will contain mathematics courses. Third, it is hypothesized that mathematics departments will have consistently high centrality in the network analysis. And finally, it is hypothesized that the interconnectedness of undergraduate curricula will be inversely correlated with the institution size.

Subjects

The subjects of this study will be the curricula of the seven universities in the Minnesota State Colleges and University System (MinnState). MinnState is the third-largest system of state colleges and universities in the United States, consisting of thirty colleges and seven universities across fifty-four campuses (Minnesota State, 2021).

The two largest universities in the MinnState system are Minnesota State University, Mankato and Saint Cloud State University. Both schools are classified as master's colleges and universities: larger programs (Carnegie Classifications, 2021). The 2020-2021 enrollment at Minnesota State University, Mankato was 17,357 while Saint Cloud State University had an enrollment of 16,326 students. Minnesota State University, Mankato has 77% of its students enrolled full-time while Saint Cloud State University has 61% of its students enrolled full-time (Minnesota State, 2021). Both schools offer undergraduate mathematics and statistics degrees, while Minnesota State University, Mankato also offers master's degrees in Mathematics and Mathematics Education (Minnesota State University Mankato, 2021; Saint Cloud State University, 2021.).

Southwest Minnesota State University in Marshall, Minnesota and Minnesota State University, Moorhead are both classified as master's colleges and universities: medium programs (Carnegie Classifications, 2021). In the 2020-2021 academic year, the enrollment at Southwest Minnesota State University was 8,718 students while Minnesota State University, Moorhead had an enrollment of 7,534 students. Southwest Minnesota State University had 29% of its students enrolled full-time while Minnesota State University, Moorhead had 69% of its students enrolled full time (Minnesota State, 2021). Both schools offer undergraduate mathematics and mathematics education degrees and graduate certificates in mathematics (Minnesota State University, Moorhead, 2021a; Minnesota State University, Moorhead, 2021b; Southwest Minnesota State University, 2021).

Winona State University and Bemidji State University are both classified as master's colleges and universities: small programs (Carnegie Classifications, 2021). The 2020-2021 enrollment at Winona State University was 8,856 students, while the enrollment at Bemidji State University was 6,354. Winona State University had 83% of its students enrolled full-time while BSU had 65% of its students enrolled full-time (Minnesota State, 2021). Both schools offer undergraduate mathematics and mathematics education degrees while Bemidji State University also offers a master’s in mathematics education (Bemidji State University, 2021; Winona State University, 2021).

The seventh university in the MinnState system is the only one classified as a doctoral/professional university (Carnegie Classifications, 2021). Metropolitan State University with campuses in Minneapolis and Saint Paul, had 10,575 students enrolled in the 2020-2021 academic year. Metropolitan State University has 40% of its students enrolled full-time and offers undergraduate degrees in mathematics and mathematics education. Metropolitan State University also offers a mathematics graduate certificate and a master's degree in urban education with an emphasis in secondary teacher preparation for mathematics teaching (Minnesota State, 2021; Metropolitan State University, 2021).

Measures

The curriculum prerequisite networks for the seven universities in the MinnState system will be constructed to test the hypotheses of this research. For each university, the prerequisite network will be built with nodes representing courses and directed edges representing prerequisite relationships.

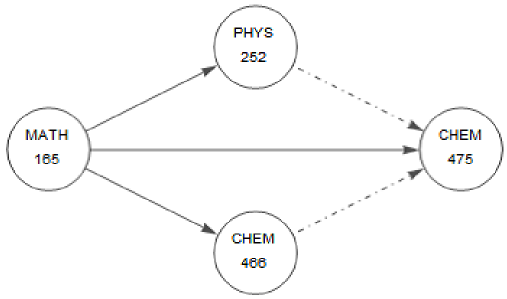
The first hypothesis states that lower-level mathematics courses will exhibit a higher variance in being labeled a prerequisite than higher level mathematics courses. Out-degree centrality will be used to test the first hypothesis. The out-degree of a node in a directed network is the number of edges that start at that node and direct to another node. Also, the out-components of the courses with high out-degree will be compared across campuses. The out-components of a node are all the nodes that lie on a directed path after the given node.

The second hypothesis states that the longest prerequisite paths through the curriculum will contain mathematics courses. The directed prerequisite networks for the universities in this study should also be acyclic. This means that there should not be a prerequisite path that starts at one class and circles back around to the same course. That is to say, no course is going to serve as its own prerequisite. If a case such as this arises, reasonable adjustments will be made to the curriculum to form an acyclic directed network. A second issue that could arise while testing this hypothesis is redundant prerequisites. A redundant prerequisite is one that is listed even though the course is also listed by another prerequisite. In Figure 4, the MATH 165 prerequisite for CHEM 475 is redundant. The MATH 165 course is a prerequisite for both PHYS 262 and CHEM 466, and one of those two courses is required to satisfy the prerequisites for CHEM 475. The MATH 165 course need not be listed as a prerequisite for CHEM 475 since it is a requirement for the other prerequisites of CHEM 475. The example represented in Figure 4 is an example of a prerequisite that comes from options. Since the prerequisites for CHEM 475 can be satisfied by PHYS 252 or CHEM 466, this is a redundant prerequisite comes from options. This research does not have a method for capturing this type of redundant prerequisite.

However, some programing in Mathematica has made it possible to locate and remove redundant prerequisites that come from course requirements that have no choices. If the example represented in Figure 4 was stated with an “and” instead of an “or” statement, then it would be a redundant prerequisite that comes from a requirement without choices. Essentially, if there is a alternate path of solid edges from the redundant prerequisite to the target course, then this research will locate and delete that redundant prerequisite edge from the network.

Figure 4

Sample of a Redundant Prerequisite



Note. MATH 165 is a redundant prerequisite for CHEM 475 since it is required for PHYS 252 and CHEM 466 and one of those is required as a prerequisite of CHEM 475.

For this research, the only paths considered between two nodes will be the geodesic paths. A geodesic path between two nodes is one that crosses the fewest number of edges while traveling from the starting node to the last node (Newman, 2010). Restricting the research to only geodesic paths comes from the idea of students trying to complete their degree in the fewest number of semesters as possible. Each node in a prerequisite path will typically indicate an additional semester of schoolwork for a student.

The third hypothesis is that mathematics departments will have consistently high centrality in the curriculum prerequisite networks. Two methods will be used to test the third hypothesis. The first method will be a visual representation of each university's prerequisite network. In this representation the nodes in the prerequisite networks will represent the academic departments on campus and the edges will represent the prerequisite connections between courses in separate departments. Using the network analysis capabilities built into *Mathematica* software (https://www.Wolfram.com/Mathematica/), an algorithm will be run to group the departments into communities based on geodesic edge betweenness centrality (Wolfram Community, 2018). The geodesic edge betweenness centrality (edge betweenness) of an edge in a network is the number of geodesic paths that contain that edge (Fortunato, 2010). The second treatment will be a measurement of course cruciality as developed by Slim et al. (2014). A course's cruciality is the sum of what Slim et al. define as the course's delay factor and a course's blocking factor. Higher delay factors for a course indicate that a failure in that course is more likely to delay a student's completion of the program. Higher blocking factors indicate that a failure in the course will block the student from progressing through more courses.

The fourth hypothesis is that the interconnectedness of undergraduate curricula will be inversely correlated with institution size. The measures used for this hypothesis will be the average out-degree of nodes in the network, the number of weakly connected components in the network, and the number of isolated nodes in the network. Two nodes in a network are in the same weakly connected component if there is a path from one node to the other. It is not possible for an acyclic directed graph to have strongly connected components. A node is isolated if it has no edges connecting it to other nodes. In the case of this research, an isolated node would represent a course that does not serve as a prerequisite and also has no prerequisites.

Design and Data Analysis

The curriculum information from the universities in this research will be scraped from each university's public website. Universities typically have an online course catalog where each course is listed with its description and any pre- or corequisites. The computer program Mathematica will be used for all of the data scraping and network analysis in this research. For universities where the data scrape yields messy data, Microsoft Excel (https://www.microsoft.com/en-us/microsoft-365/excel) will be used to clean up the data into a usable format.

In the preparation of the curriculum data for each university, corequisites will be treated the same as prerequisites with the exception of two-way corequisites. In the case of two-way corequisites, the two courses will be listed as a single course with a single node in the network analysis. For example, if BIOL 105 has a corequisite of the lab course BIOL 105L and vice versa, then the course will be listed as BIOL 105/BIOL 105L. Also, prerequisites that do not refer to a particular course will not be a part of this research. This exclusion includes prerequisites with wording similar to Consent of Instructor, Minimum of 3.5 GPA, 15 Completed Credit Hours, Previous Experience in Programming, Admission to the Program, or Department Majors Declared. Additionally, any prerequisite references to recommended courses, high school courses, or non-college credit courses will not be included in this research. Finally, statements of minimum required grades for a prerequisite course will not be included. For example, a prerequisite of MATH 107 with a grade of C or higher will be treated as just a prerequisite of MATH 107.

The cleaned data indicating each prerequisite connection and its weight will then be entered back into Mathematica and a network graph will be produced using the software. The curriculum data for each university should lead to an acyclic directed network. If there are cycles in the resulting network, reasonable accommodations to make the network acyclic will be made at the discretion of the researcher.

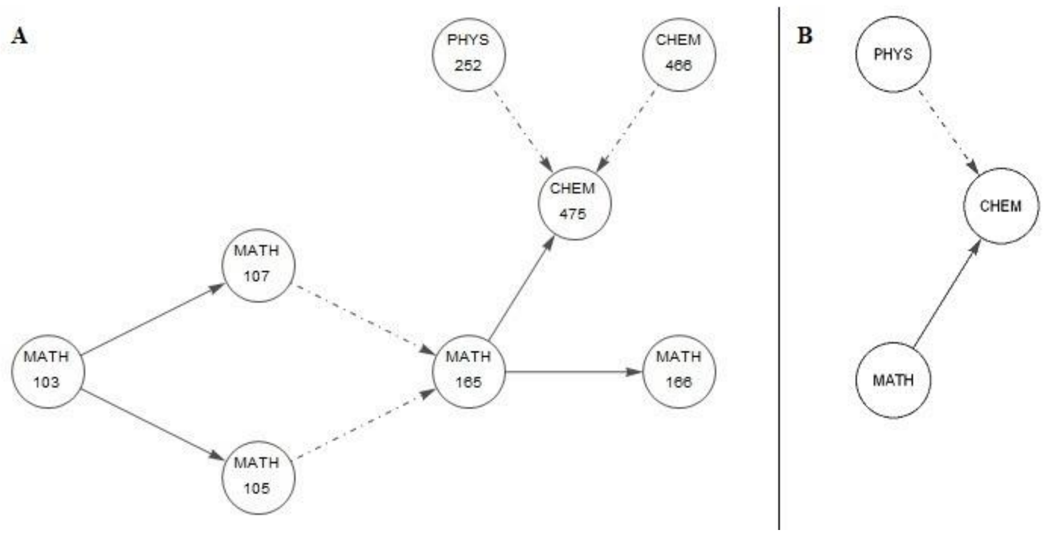
The weighting of the edges in the prerequisite network will vary by the available choices listed in the prerequisite statement. If a single course is required without any alternative choice, the edge representing that connection will be weighted as one. If one of *n* courses can satisfy the prerequisite, then each of those two edges will have a weight of 1/*n* (see Figure 1).

The departmental prerequisite network will be used to examine the interactions between departments based on prerequisites. In this type of network, all intradepartmental prerequisites will be removed from the data. The remaining prerequisites will then be redefined only using department names and excluding course numbers. For example, the relation where CHEM 475 calls MATH 165 as a prerequisite will instead be defined as CHEM requiring MATH as a prerequisite. This could make for several repetitive edges from one department to another, although the edges could have different weights. The analysis will use one edge representing the sum of all the weights between two departments (see Figure 5).

The algorithms for calculating the out-degree centrality for the testing of hypothesis one, the network graph in the testing of hypothesis three, and finding weakly connected components for the testing of hypothesis four are built in commands for Mathematica. The longest path calculation for the testing of hypothesis two as well as the blocking factor and delay factor in the testing of hypothesis three require some additional programming in Mathematica.

Figure 5

Conversion from a Prerequisite Network to the Department Network



Note. Panel A has a sample prerequisite network and Panel B has the resulting department network from the same information.

The process used to calculate the longest paths in the testing of hypothesis two will be used to help calculate the delay factor. The delay factor will be used in the testing of hypothesis three. To evaluate the delay factor of a node, a built-in command in the Mathematica software will be used to build a matrix that gives the geodesic path length between every pair of nodes in the prerequisite network. The maximum of these values will be the longest geodesic paths in the network and pulling the courses that make up the paths of maximal length from the network information is again a built-in command of the Mathematica software. Reviewing these paths for mathematics courses will be the test for hypothesis two. The delay factor for a node in a prerequisite network is defined as the length of the longest geodesic path that contains that node (Slim et al., 2014). The distance matrix in the process listed above will be used to calculate the delay factor for each node in the network. From this information and the lists of courses that comprise each path, a collection of nested proper subsets can be formed so that the smallest subset that contains a course is also the course's delay factor.

The blocking factor calculation for each course in the prerequisite network is a quick calculation in Mathematica. The blocking factor for a course in a prerequisite network is the number of unique courses, excluding itself, that exist in all possible paths starting at that course (Slim et al., 2014). A command named VertexOutComponent in Mathematica lists the original course and all of the courses that define its blocking factor. Therefore, the blocking factor for any course in a prerequisite network will be the value of its VertexOutComponent minus one. The cruciality factor for a course will be used to test hypothesis three. The cruciality of a course is the sum of its blocking and delay factors. The complexity of a department is then the sum of the cruciality factors for all of the courses in the department.

Chapter IV - Results

This research built and used the curriculum prerequisite networks for the universities in the Minnesota State (MinnState) system to compare curriculum structures. The prerequisite information for each course was taken from the course catalogs available on each university's public website. The data collection for this research took place in the fall of 2021. The Wolfram Mathematica software (https://www.wolfram.com/mathematica/) was used to collect all of the data. Mathematica was also used for all computations and graph rendering throughout this research.

The MinnState system is comprised of thirty colleges and seven universities across fifty-four campuses (Minnesota State, 2021). The fall 2021 enrollments and Carnegie university classifications (Carnegie Classifications, 2021) for each university can be found in Table 1. The Carnegie university classification for each school was largely based on the number of degrees awarded during the 2019-2020 academic year. For example, the classification of Metropolitan State University as a Doctoral/Professional University indicated that at least 30 professional practice doctoral degrees in at least two programs were awarded in 2019-2020. All of the other universities are classified as Master's Granting institutions. The large, medium, and small classifications indicate that more than 200, 100 to 199, or less than 100 master's degrees respectively were awarded at the institution in 2019-2020 (Carnegie Classifications, 2021).

Before the hypotheses of this research are addressed, it is important to first inspect some of the high-level features of the MinnState university system. The following information will describe the size and distribution of features in the curriculum prerequisite networks for the universities in the MinnState system and thus can help define a starting point for how to evaluate the hypotheses. The initial metrics of student enrollment in the fall of 2021 (Minnesota State, 2021), the total number of courses in each university's online course catalog, the distribution of isolated, and linked courses in each network are reported in Table 1. Isolated courses had no prerequisites and did not serve as the prerequisites for any other course. These courses were not connected to any edges in the network and had a total in-degree and out-degree of zero. Linked courses either served as a prerequisite for at least one course or had at least one prerequisite. Linked courses were connected to at least one edge in the network and had a total in and out-degree greater than zero. The data reported throughout this research will reflect the courses and prerequisites that have been adjusted from the initial online course catalog listings to accommodate the structure of curriculum prerequisite networks.

Since the networks used in this research needed to be acyclic and were built only to consider those requirements that listed specific courses as prerequisites, changes needed to be made to some prerequisite listings in each university's online course catalog. Changes that were made to the online course catalog listings included combining corequisite courses into a single course, removing prerequisites that did not call a specific course or courses, and removing redundant prerequisites that have no alternatives. Corequisite courses were listed as a single course node in the network if both courses were required to be taken at the same time. This was the case with many lab courses. Additionally, courses were listed as a single course node in the network if both courses required the other as a prerequisite. Examples of requirements that were removed because they did not call a specific course include statements that require acceptance into a major or program, previous attainment of a certain number of general or in-program credits, a passing grade of “C” or higher, and permission from a department or person. Additionally, it was found that some courses listed prerequisite courses that no longer existed in the current course catalog. These courses that no longer existed were deleted as prerequisites and appropriately substitutes were added when the target course listing could be found on the MinnState course registration system.

After all of the data collection and adjustments to each university’s course catalog were complete, five percent of each university’s collected course information was audited with a comparison to the online catalog to assure accuracy. This auditing process was repeated for each university until no errors were found in a five percent audit.

The number of courses found in the curricula for each university in the MinnState system and the distribution of those courses into linked and isolated courses showed some consistent patterns. The two largest universities in the system, Minnesota State University, Mankato and St. Cloud State University, had the largest number of isolated, linked, and total courses in their curricula as well as the smallest percentage of linked courses (see Table 1). The two universities which were classified as Master’s Granting Institutions: Medium Size, Southwest Minnesota State University and Minnesota State University, Moorhead, also shared similar demographics for all of the categories in Table 1. Two universities were classified as Master’s Granting Institutions: Small Size, Winona State University and Bemidji State University. These universities had similar numbers of courses in their catalog as the medium-sized institutions, but the small institutions had the largest percentage of linked courses across the system. Metropolitan State University was the only university classified as a Doctoral/Professional Granting Institution in the MinnState university system. The metrics of linked, isolated, and total courses in the Metropolitan State University curriculum were very similar to those of the small institutions.

Table 1

Fall 2021 Classification, Enrollment, and Course Listings for MinnState Universities

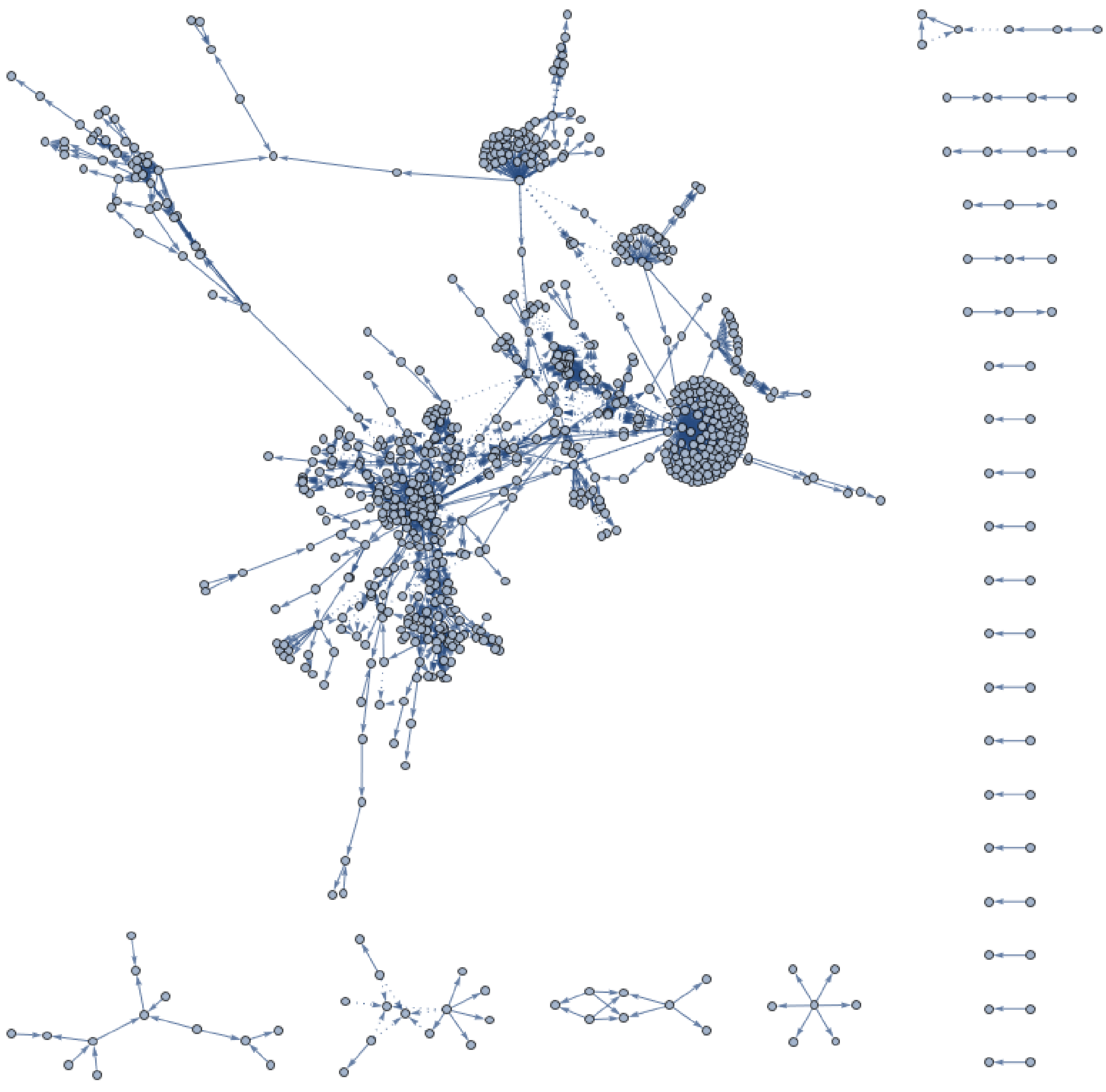
Carnegie Classification	University	Student Enrollment	Isolated Courses	Linked Courses	Total Courses	% Linked Courses
Doctoral/ Professional University	Metropolitan State	10,575	682	795	1,477	53.8%
Master's University: Large	Minnesota State, Mankato	17,357	1,668	1,192	2,860	41.7%
	St. Cloud State	16,326	2,082	1,166	3,248	35.9%
Master's University: Medium	Southwest Minnesota State	8,718	769	592	1,361	43.5%
	Minnesota State, Moorhead	7,534	1,027	761	1,788	42.6%
Master's University: Small	Winona State	8,856	626	1,138	1,764	64.5%
	Bemidji State	6,354	612	743	1,358	55.0%

Note: The sum of the in-degree and out-degree of an isolated course is zero. A linked course has a total degree that is more than zero.

The curriculum prerequisite network for Metropolitan State University was made up of 795 linked courses separated into several components (see Figure 6). In an acyclic directed network, components of linked courses are defined to be weakly connected. All of the networks in the MinnState system were acyclic-directed networks. Acyclic-directed networks cannot be considered strongly connected (connected). The largest weakly connected component in the Metropolitan State University network was

Figure 6

The Curriculum Prerequisite Network for the Metropolitan State University



Note: The Metropolitan State University Prerequisite network had 795 linked courses and 682 isolated courses (not shown). See Appendix A for more information on the departmental makeup of components.

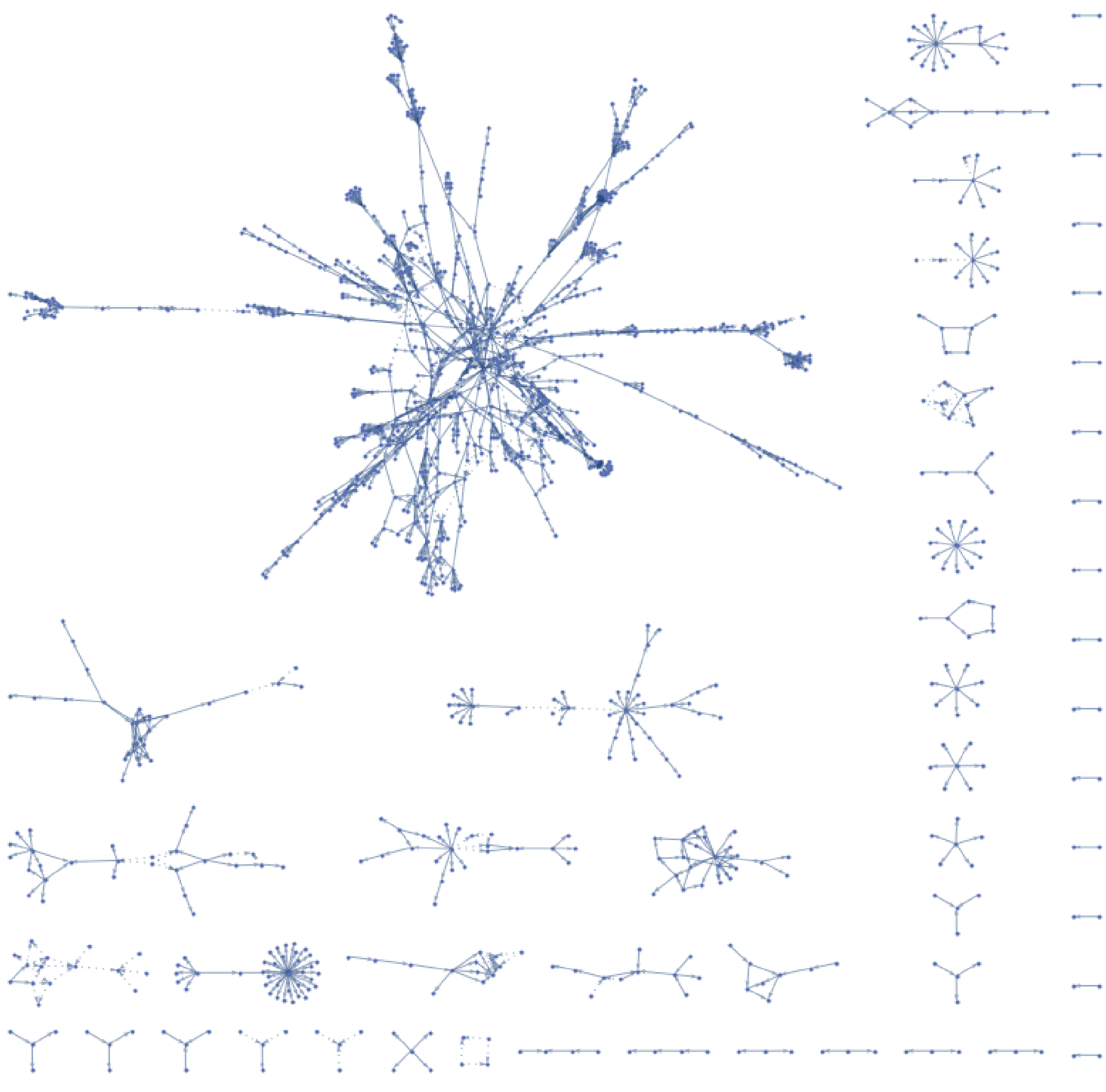
composed of 703 courses. The four smaller components at the bottom of Figure 6 show, from left to right, courses from the departments of nursing, art, human services, and human resource management. See Appendix A for more information on the composition of courses in the different network components. Overall, the network contained 2 components with 11 to 25 courses, 3 components with 6 to 10 courses, and 19 components with 5 or fewer courses. This was by far the smallest number of components for any curriculum prerequisite network in the MinnState university system. The large component in the network accounted for 88.4% of all the linked courses in the network and 47.6% of all courses in the curriculum (see Table 2). The most predominant feature of the largest component is a cluster of 170 courses that all contained WRIT 131: Writing 1 as a prerequisite.

The curriculum prerequisite network for Minnesota State University, Mankato contained 1,192 courses separated into several components (see Figure 7). The network consisted of one large component composed of 727 courses and several smaller components that vary in size from 43 to 2 courses. Specifically, there were 3 components with 26 to 50 courses, 10 smaller components that contain 11 to 25 courses, 8 components with 6 to 10 courses, and 49 components with 5 or fewer courses (see Table 2). Refer to Appendix B for more information on the courses that were found in the different weakly connected components. Across the entire MinnState university system, Minnesota State University, Mankato had the most components with six or more courses. The largest component in the network accounted for 60.0% of all the linked courses in the network and 25.4% of all courses in the curriculum (see Table 2).

For St. Cloud State University, the curriculum prerequisite network was composed of 1,166 linked coursed and 2,082 isolated courses (see Figure 8). The network had one large weakly connected component composed of 717 courses and many smaller

Figure 7

The Curriculum Prerequisite Network for Minnesota State University, Mankato

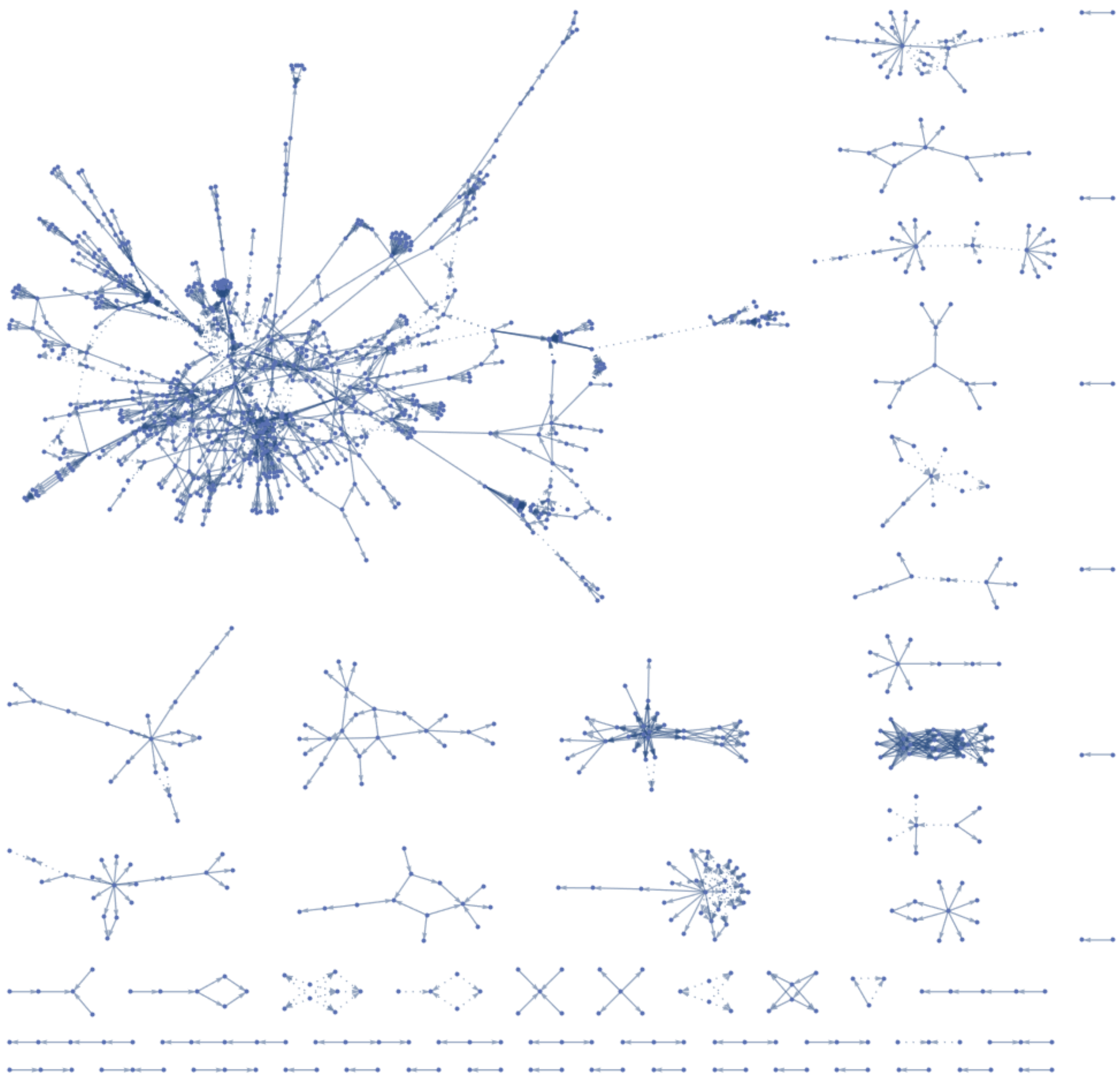


Note: The Minnesota State University, Mankato prerequisite network had 1,192 linked courses and 1,668 isolated courses (not shown). See Appendix A for more information on the departmental makeup of components.

components. The large component in the network accounted for 61.4% of all the linked courses in the network and 22.1% of all courses in the curriculum. St. Cloud State University network had the largest number of components across all of the universities in

Figure 8

The Curriculum Prerequisite Network for St. Cloud State University

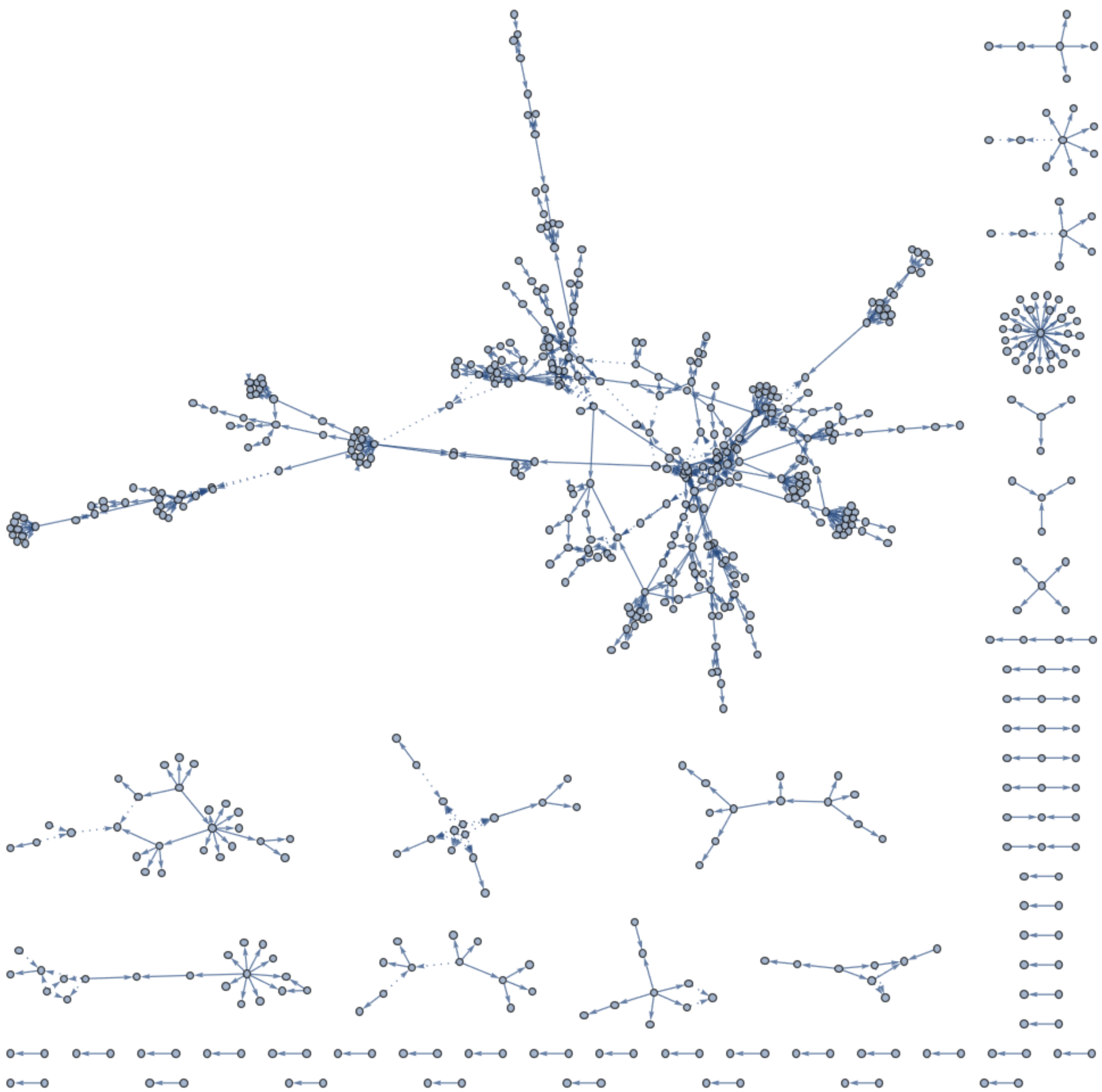


Note: The St. Cloud State University prerequisite network had 2,082 linked courses and 1,166 isolated courses (not shown). See Appendix C for more information on the departmental makeup of components.

the MinnState system. This included having the most components, 59, with five or fewer courses (see Table 2). The second through fourth largest components contained coursed from the art, nursing, and criminal justice departments (see Appendix C).

Figure 9

The Curriculum Prerequisite Network for Southwest Minnesota State University



Note: The Southwest Minnesota State University prerequisite network had 592 linked courses and 769 isolated courses (not shown). See Appendix D for more information on the departmental makeup of components.

Figure 9 show the curriculum prerequisite network for Southwest Minnesota State University excluding the 769 isolated course. The network's largest component was composed of 339 courses. This was the smallest number of courses in the largest component for any of the schools in the MinnState University system. The next two largest components are both composed of courses from the music department (see Appendix D). The large component in the network accounted for 56.8% of all the linked courses in the network and 24.7% of all courses in the curriculum (see Table 2).

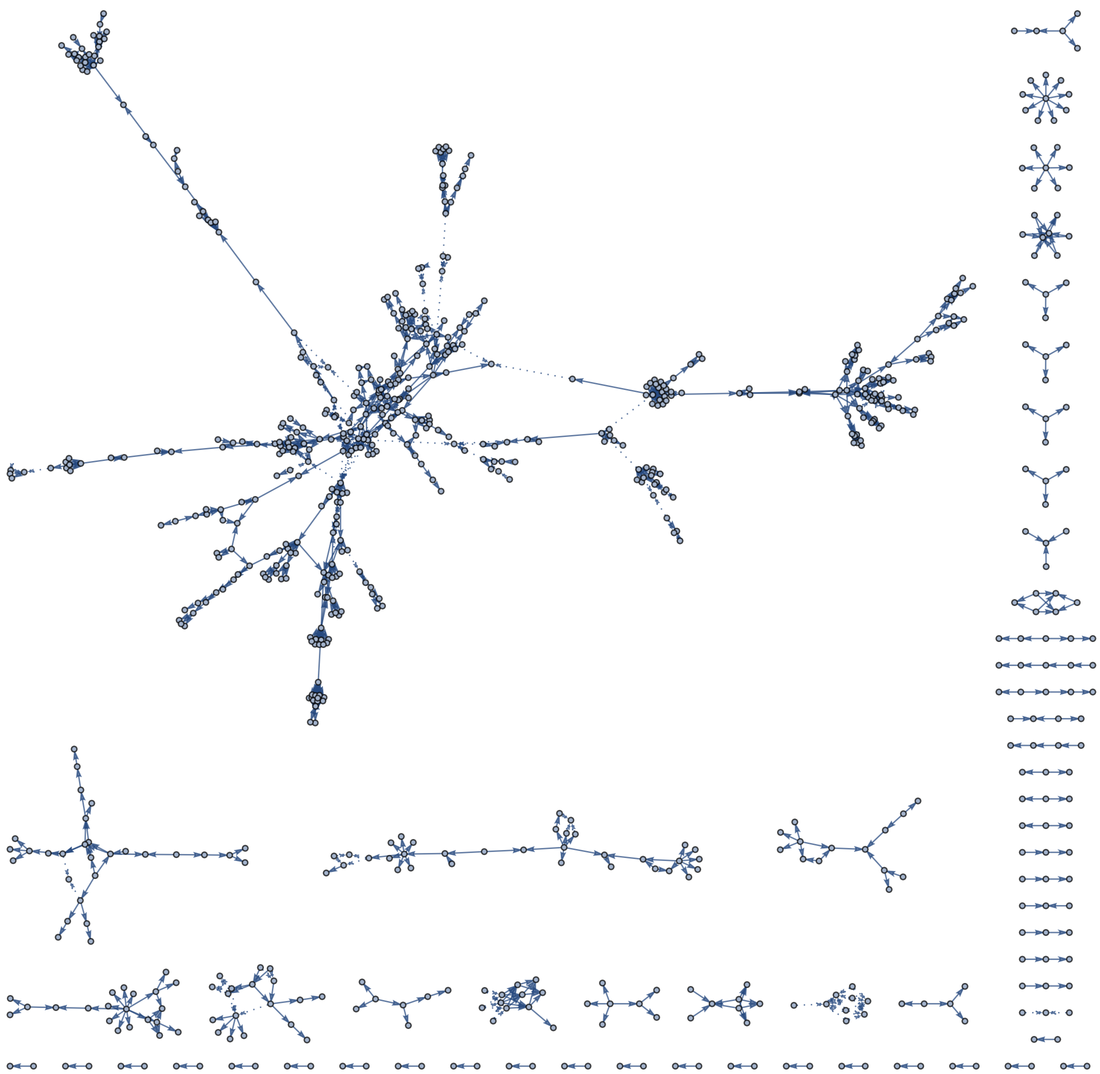
Figure 10 shows the curriculum prerequisite network for Minnesota State University, Moorhead excluding the 1,027 isolated courses. The largest component in the network was composed of 474 courses (see Table 2). The next two largest components are in the row below the largest component. The component on the left is composed of courses from the music department and the one in the middle is composed of courses from the photography, film, and animation departments (see Appendix E).

Figure 11 shows the curriculum prerequisite network for Winona State University excluding the 626 isolated courses. The network consisted of one large weakly connected component composed of 868 courses, 5 smaller components that contain 11 to 25 courses, 12 components with 6 to 10 courses, and 40 components with 5 or fewer courses. The large component in the Winona State University curriculum prerequisite network was the largest component across all of the networks representing universities in the MinnState system. Also, only the networks of Winona State University and Metropolitan State University had no components with twenty-six to fifty courses. The large component in the network accounted for 76.3% of all the linked courses in the network and 49.2% of all courses in the curriculum (see Table 2).

Figure 12 shows the curriculum prerequisite network for Bemidji State University excluding the 612 isolated courses. The network consisted of one large weakly connected component composed of 403 courses, 1 medium-sized component of 93 courses, 2 small

Figure 10

The Curriculum Prerequisite Network for Minnesota State University, Moorhead

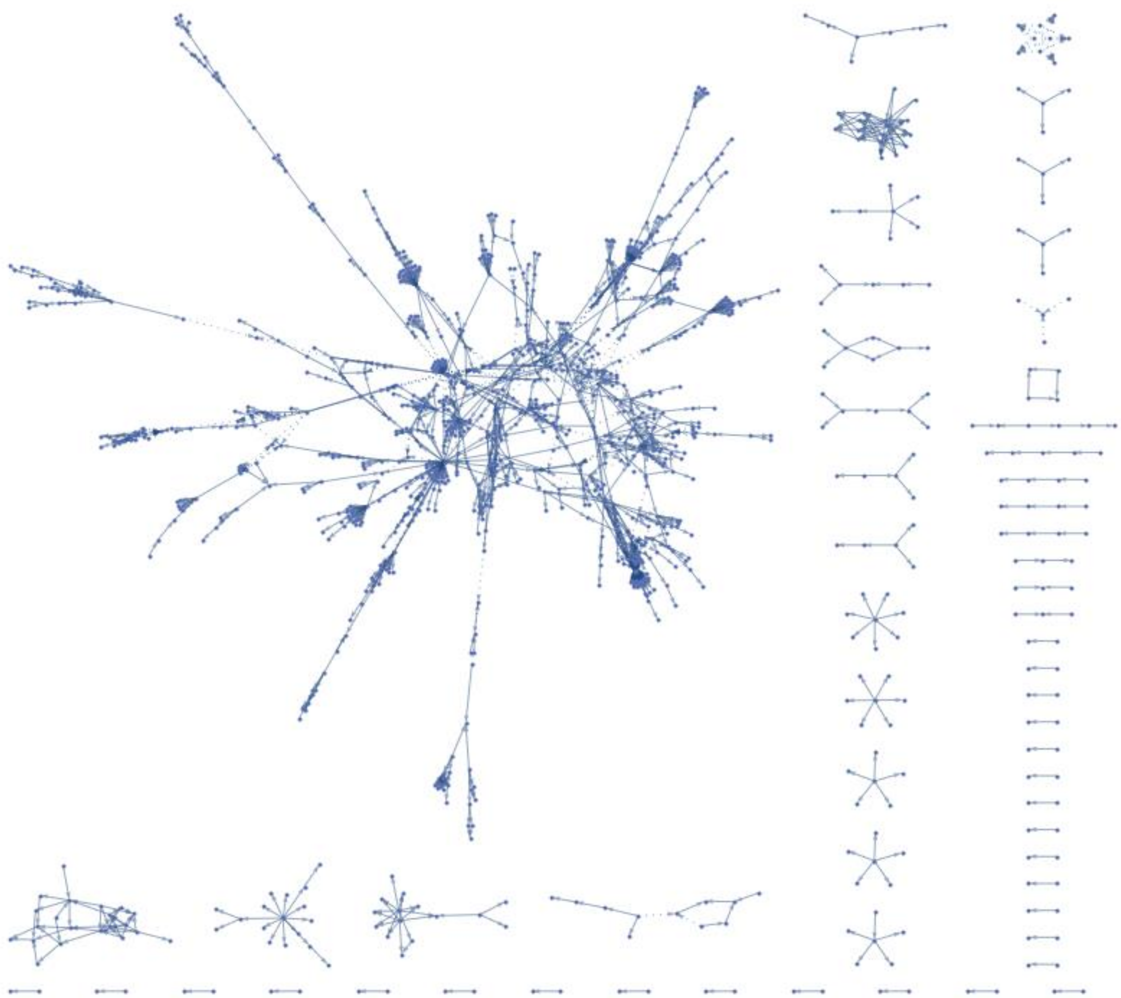


Note: The Minnesota State University, Moorhead prerequisite network had 761 linked courses and 1,027 isolated courses (not shown). See Appendix E for more information on the departmental makeup of components.

components that contain 26 to 50 courses, 3 smaller components that contain 11 to 25 courses, 3 components with 6 to 10 courses, and 42 components with 5 or fewer courses. This was the only network in the MinnState university system that had its second-largest component made up of more than fifty courses (see Table 2). The second-largest

Figure 11

The Curriculum Prerequisite Network for Winona State University

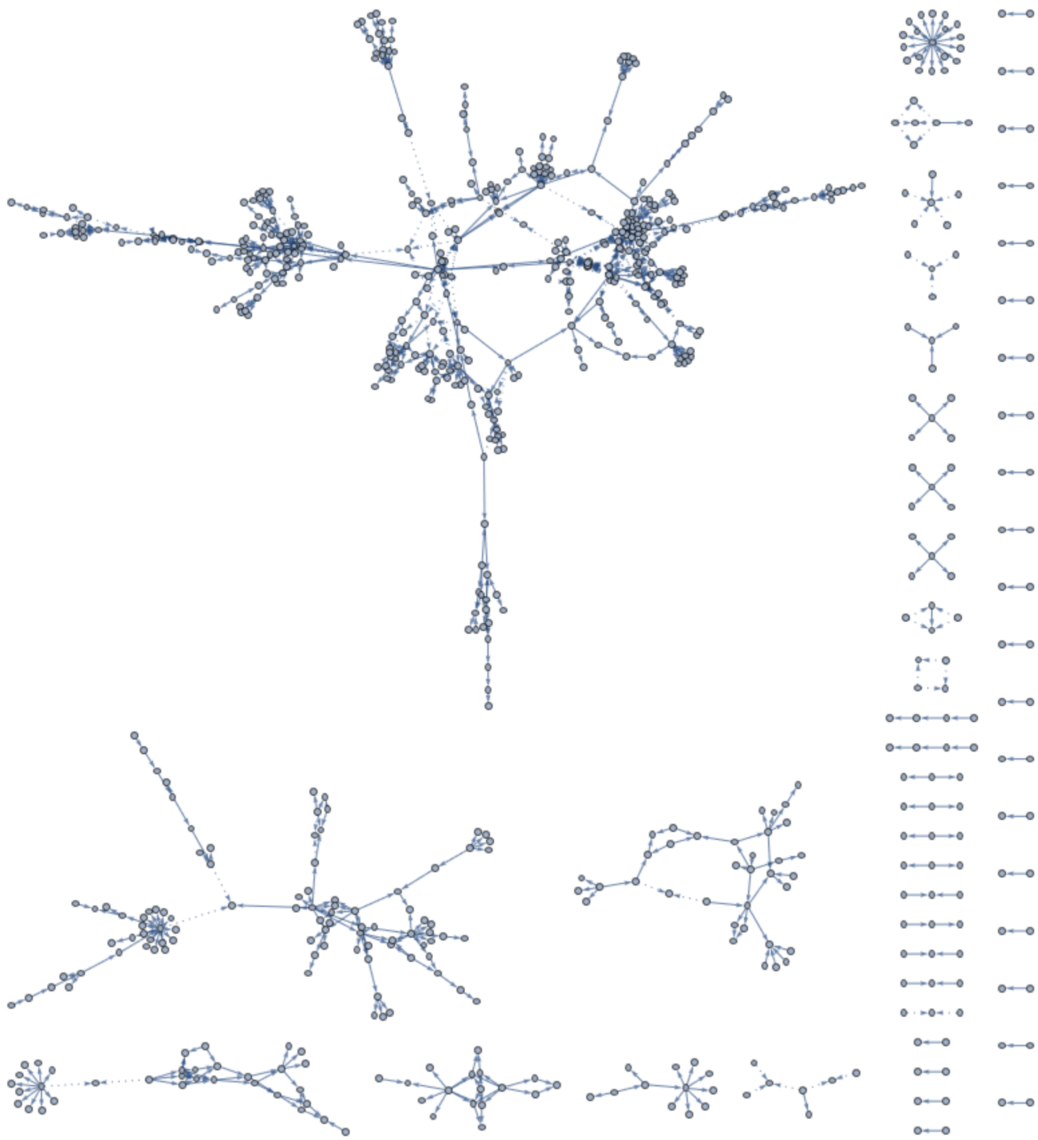


Note: The Winona State University prerequisite network had 626 linked courses and 1,138 isolated courses (not shown). See Appendix F for more information on the departmental makeup of components.

component in this network was composed of courses from the education, math, modern languages, Ojibwe, physical education, Spanish, and special education departments. See

Figure 12

The Curriculum Prerequisite Network for Bemidji State University



Note: The Bemidji State University prerequisite network had 743 linked courses and 612 isolated courses (not shown). See Appendix G for more information on the departmental makeup of components.

Appendix G for more information on the courses that exist in the weakly connected components of Figure 12. The large component in the network accounted for 82.4% of all the linked courses in the network and 45.1% of all courses in the curriculum.

Table 2

Distribution of Component Size in MinnState University Curriculum Prerequisite Maps

Carnegie University Classification	Weakly Connected Components with the Indicated Number of Courses					Number of Courses in the Largest Component	% of Linked Courses in Largest Component	% of All Courses in Largest Component
	2-5	6-10	11-25	26-50	51-99			
Doctoral/ Professional								
Metropolitan	19	3	2	0	0	703	88.4%	47.6%
Master's: Large								
Mankato	49	8	10	3	0	727	60.0%	25.4%
St. Cloud	59	7	9	2	0	717	61.4%	22.1%
Master's: Medium								
Southwest	42	5	4	2	0	339	56.8%	24.7%
Moorhead	43	8	4	2	0	474	62.3%	26.5%
Master's: Small								
Winona	40	12	5	0	0	868	76.3%	49.2%
Bemidji	42	3	3	2	1	403	82.4%	45.1%

The distribution for the different sizes of weakly connected components across all of the universities in the MinnState system is found in Table 2. The Master's Granting Institutions of large size appeared to have similar demographics and the same can be said for Master's Granting Institutions of medium size. The Winona State University and Bemidji State University networks had similar percentages of courses devoted to their largest weakly connected components. They had about the same number of components with one to five courses, but the rest of the component distribution did not appear to be similar.

Hypothesis 1

The first hypothesis of this research was that lower-level courses in mathematics departments would exhibit a higher variance in being listed a prerequisite than higher-level courses. The out degrees of similar mathematics courses across the MinnState university system were inconsistent (see Table 3). Similar mathematics courses across the different universities in the MinnState university system were determined using the Transferology.com website. Transferology (https://www.transferology.com/) is a public site that indicates how credits may transfer from one university to another.

In Table 3, each course is a predicted credit equivalency on Transferology. Although some equivalencies take multiple steps. For example, Transferology predicted that Minnesota State University, Mankato Discrete Mathematics course would only transfer as elective credits to St. Cloud State University. However, Transferology predicted that both the Minnesota State University, Mankato and St. Cloud State University Discrete Mathematics courses would have directly transferred to Metropolitan State University as Discrete Mathematics credits (Transferology, 2021).

As seen in Table 3, the number of courses at a given university that listed a specific mathematics course as a prerequisite varied drastically in the MinnState System. Metropolitan State University had 23 courses that listed the pre-college level Intermediate Algebra course as a prerequisite while Saint Cloud State University only had 14 such courses. The rest of the universities in the MinnState system averaged only about 5.2 courses that listed Intermediate Algebra as a prerequisite. Similarly, Metropolitan State University had 65 courses that listed College Algebra as a prerequisite while Winona State University had 3. The other five universities in the MinnState

Table 3

Out-Degree of Similar Courses in MinnState University Mathematics Departments

	Metro.	Mankato	St. Cloud	SMSU	Moorhead	Winona	Bemidji
Int. Algebra	23	5	14	4	4	8	5
College Algebra	65	19	22	16	9	3	14
Trigonometry	--	9	5	2	2	--	4
Precalculus	4	1	12	8	2	8	6
Calculus I	10	24	16	1	7	20	4
Calculus II	9	14	31	13	10	12	11
Calculus III	3	14	9	2	7	4	2
Discrete Math.	13	3	11	2	6	1	9
Linear Algebra	5	8	9	0	4	3	2
Differential Equ.	3	9	5	1	5	11	2
Real Analysis	1	1	1	1	1	2	0
Abstract Algebra	0	7	1	1	1	1	0

Note: Metro.= Metropolitan State University, SMSU= Southwest Minnesota State University, Int. Algebra= Intermediate Algebra, Discrete Math.=Discrete Mathematics, Differential Equ.=Differential Equations. Dashes indicate a course that was not in the university's curriculum. See Appendix H for specific course numbers.

university system averaged about 17 courses that listed College Algebra as a prerequisite. For Calculus I, Minnesota State University, Mankato had 24 courses that listed it as a prerequisite while Southwest Minnesota State University had only 1. The higher-level courses of Calculus III, Discrete Mathematics, Linear Algebra, Differential Equations, and Abstract Algebra had similar but less extreme variances in the number of times they were listed as prerequisites.

Hypothesis 2

The second hypothesis of this research was that the longest geodesic paths in the curriculum prerequisite networks for the universities in the MinnState system would contain mathematics courses. The longest geodesic paths in the networks for Minnesota State University, Mankato, Southwest Minnesota State University, Minnesota State

University, Moorhead, and Winona State University all contained mathematics courses (see Tables 4-5). The only three mathematics courses in the MinnState system that had prerequisites options outside of the mathematics department were from St. Cloud State University and Winona State University. These courses were only involved with 16 of the longest geodesic paths at Winona State University and one similar path at St. Cloud State University. Mathematics courses existed in and began the longest geodesic paths for the majority of long paths at every university except Bemidji State University. Similarly, outside of Bemidji State University, each university had a steady increase in the percent of paths that contained mathematics courses as the path lengths also increased.

Table 4

Longest Geodesic Paths at the Three Largest MinnState Universities

Courses	Metropolitan		Mankato		St. Cloud	
in Path	Count	%	Count	%	Count	%
2	489	5%	555	1%	670	1%
3	599	21%	464	12%	552	4%
4	324	43%	540	29%	1254	10%
5	178	69%	613	63%	3480	10%
6	42	93%	268	85%	630	86%
7	14	93%	119	93%	285	91%
8	2	50%	24	92%	54	91%
9	1	0%	8	100%		
10	1	0%	4	100%		
Total		28%		38%		20%

Note: Count refers to the number of paths that were of the indicated length and % indicates the percentage of those paths that contained at least one mathematics course. This information is continued in Table 5.

Hypothesis 3

The third hypothesis of this research stated that mathematics departments would have consistently high centrality in the university curriculum prerequisite networks.

To investigate this hypothesis, the departmental curriculum prerequisite networks (see Figure 5) were constructed for each university in the MinnState university system. Additionally, the department nodes in the departmental networks were also clustered into communities based on the strength of their connections. The Mathematica software was used to find network communities using a method based on edge betweenness centrality. The edges in the departmental network were weighted based upon the number of prerequisite connections and the weight of those connections between departments. When departments were clustered together, it indicated that there were multiple prerequisite relationships between the departments in the cluster.

Table 5

Longest Geodesic Paths at the Four Smallest MinnState Universities

Courses	Southwest		Moorhead		Winona		Bemidji	
in Path	Count	%	Count	%	Count	%	Count	%
2	413	10%	399	4%	681	2%	425	2%
3	212	50%	293	13%	724	5%	328	8%
4	155	79%	381	24%	452	20%	195	28%
5	55	95%	284	67%	263	50%	215	54%
6	10	100%	287	91%	321	79%	106	50%
7	2	100%	203	97%	141	87%	55	38%
8			65	100%	23	91%	3	0%
9			6	100%	4	100%		
10								
Total		39%		45%		25%		21%

Note: Count refers to the number of paths that were of the indicated length and % indicates the percentage of those paths that contained at least one mathematics course. This information is a continuation of Table 4.

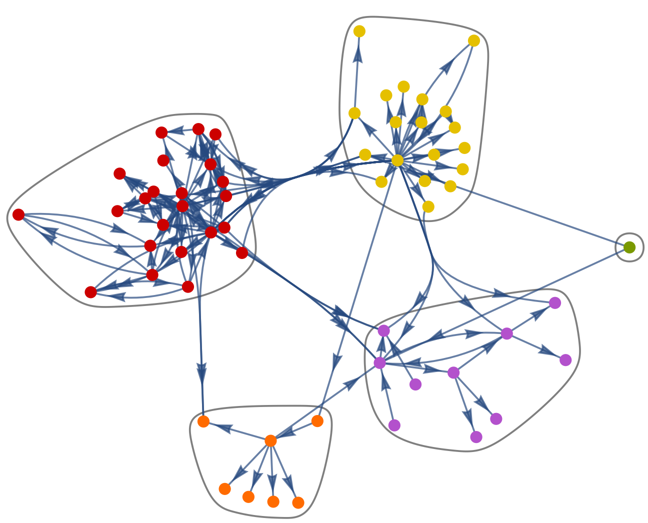
In the department prerequisite network for Metropolitan State University, the mathematics department was grouped in a cluster with twenty-three other departments. This cluster is at the top left side of Figure 13. There did not appear to be one cluster of departments that is more central to the departmental network than the others. However, the cluster containing the mathematics department was the largest of the five clusters in the network.

The department prerequisite network for Minnesota State University, Mankato had a clear central cluster of departments that includes mathematics, aviation, computer science, electrical engineering, English, physics, and four additional departments. While this largest central cluster was composed of eleven departments, no other cluster contained more than six departments (see Figure 14). See Appendix J for the departments in each of the clusters shown in Figure 14.

The departmental prerequisite network for St. Cloud State University had a clear central cluster of departments that contained the departments of mathematics, biology,

Figure 13

Departmental Prerequisite Network for Metropolitan State University

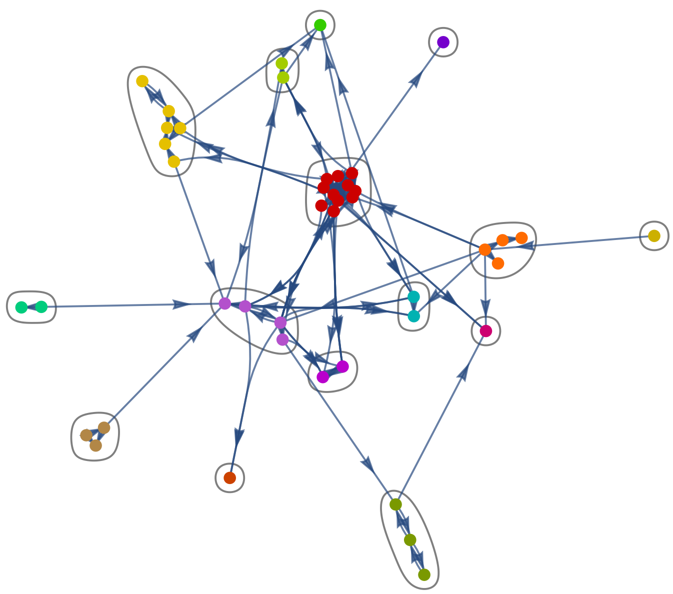


Note: The Metropolitan State University departmental network had 65 linked departments and 6 isolated departments (not shown). See Appendix J for the departments in each cluster.

chemistry, computer science, engineering, physics, economics, geography, and eleven other departments (see Appendix K). The network also had department clusters that contained eight, seven, and six departments. There were additionally ten smaller clusters that each contained one to four departments (see Figure 15).

Figure 14

Departmental Prerequisite Network for Minnesota State University, Mankato

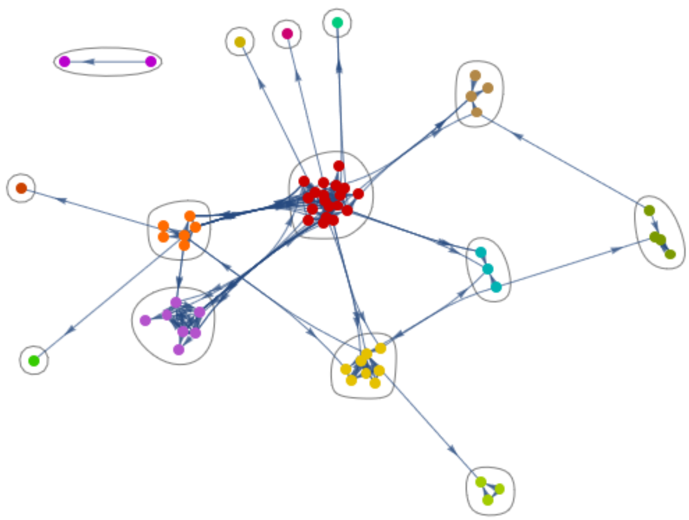


Note: The Minnesota State University, Mankato departmental network had 67 linked departments and 15 isolated departments (not shown). See Appendix J for the departments in each cluster.

The departmental prerequisite network for Southwest Minnesota State University did not have a clear central clustering of departments (see Figure 16). The mathematics department was in a cluster with the accounting, agribusiness, data science, finance, computer science, economics, and management departments. This is the center-left cluster in Figure 16. See Appendix L for more information on the departments that were contained in each of the clusters in Figure 16. The mathematics department was not

Figure 15

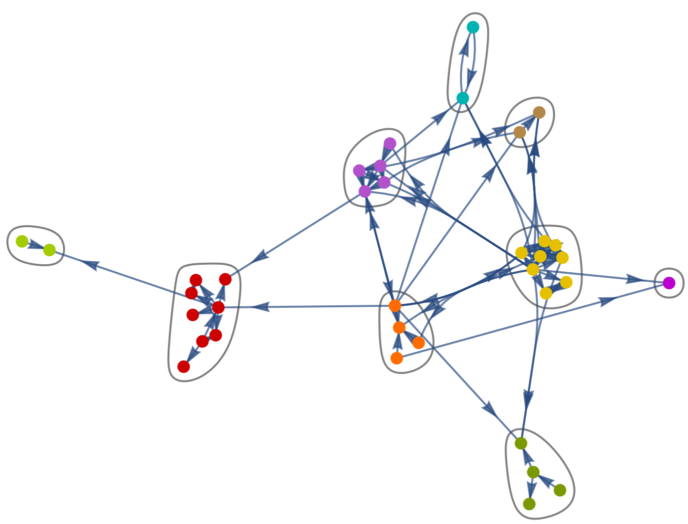
Departmental Prerequisite Network for St. Cloud State University



Note: The St. Cloud State University departmental network has 76 linked departments and 23 isolated departments (not shown). See Appendix K for the departments in each cluster.

Figure 16

Departmental Prerequisite Network for Southwest Minnesota State University



Note: The Southwest Minnesota State University departmental network had 43 linked departments and 9 isolated departments (not shown). See Appendix L for the departments in each cluster.

clustered with the other traditional STEM departments in the St. Cloud State University departmental network.

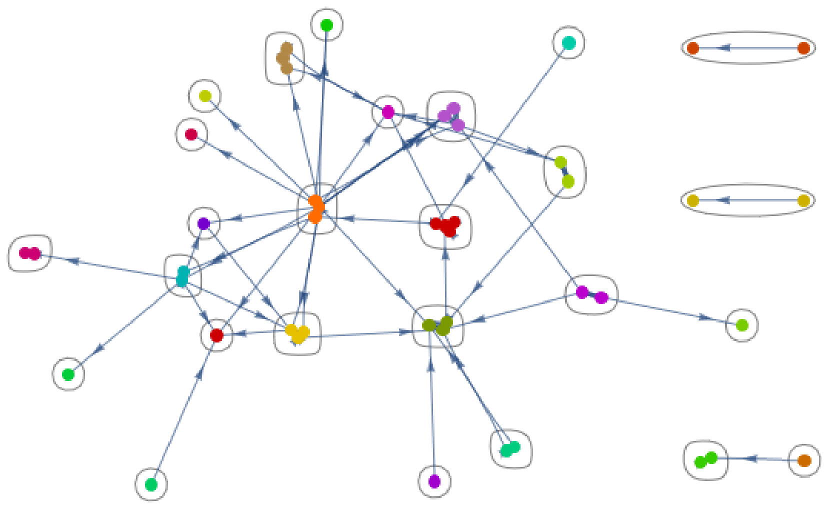
The departmental prerequisite network for Minnesota State University, Moorhead had only one cluster with four departments and five clusters with three departments (see Figure 17). There were additionally eight clusters with two courses and twelve with only one linked course. The cluster of three departments that appeared to be the most central to the graph contained the departments of computer science and information systems, mathematics, and physics (see Appendix M). Due to the abundance of small clusters, the departmental network for Minnesota State University, Moorhead appears to be similar to that of Winona State University (see Figure 18). Additionally, many of the departmental networks that had small clusters did not have the mathematics department grouped with traditional STEM departments.

The departmental prerequisite network for Winona State University appeared to be one of the least clustered graphs in the MinnState system (see Figure 18). Twenty-one of the networks twenty-four clusters had four or fewer departments and fifteen of those contained only one department. There is a triangle of three clusters, one cluster with seven departments and two with six departments, which appeared to be central to the network. The cluster containing the mathematics department was on the left side of this triangle of clusters (see Figure 18). Grouped with the mathematics department were the departments of computer science, physics, composite materials engineering, communication studies, data science, and statistics (see Appendix N).

The departmental network for Bemidji State University did not appear to have a clear central cluster of departments (see Figure 19). This was the only network in the

Figure 17

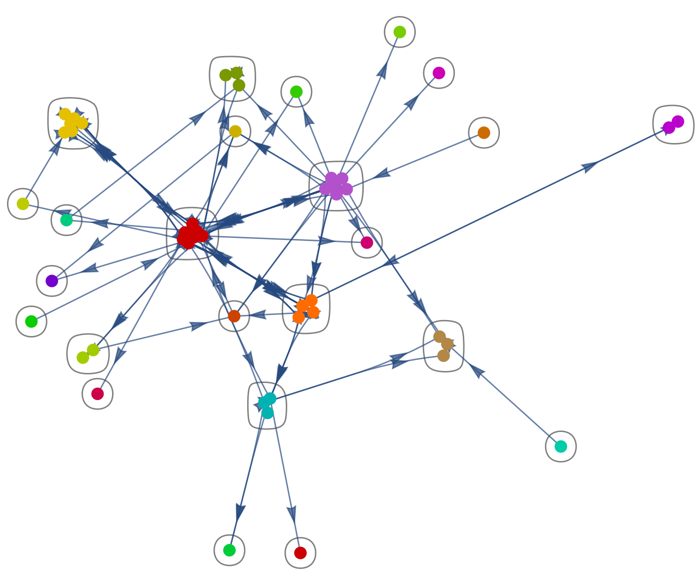
Departmental Prerequisite Network for Minnesota State University, Moorhead



Note: The Minnesota State University, Moorhead departmental network had 57 linked departments and 12 isolated departments (not shown). See Appendix M for the departments in each cluster.

Figure 18

Departmental Prerequisite Network for Winona State University

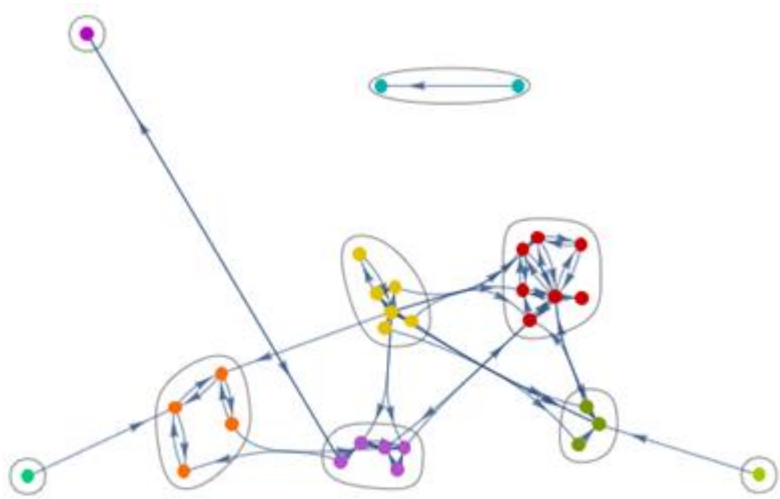


Note: The Winona State University departmental network had 60 linked departments and 7 isolated departments (not shown). See Appendix N for the departments in each cluster.

MinnState system that did not have the mathematics department in the largest cluster of the network. The mathematics department was in a cluster with the computer science, physics, sociology, statistics, and technology, and the art and design departments. The cluster with the mathematics department is located in the top middle portion of the large weakly connected component in Figure 19. See Appendix O for more information on the departmental composition of clusters in figure 19.

Figure 19

Departmental Prerequisite Network for Bemidji State University



Note: The Bemidji State University departmental network had 43 linked departments and 4 isolated departments (not shown). See Appendix O for the departments in each cluster.

Slim et. al (2014) developed the centrality measure of cruciality for curriculum prerequisite networks. The measure of cruciality for a course in a curriculum prerequisite network is the sum of the longest geodesic path that contains the course plus the size of the course's out component. The out-component of a course contains all of the courses that lie on a path after the given course. Slim et. al defined these values as the *delay* and *blocking factors* of a course. The *delay factor* is so named due to the likelihood of a delay in graduating due to a lack of success in a course that lies in long prerequisite paths. The *blocking factor* is so named because all courses in the out-component are blocked from student enrollment until success is achieved in the original course.

The prerequisites for mathematics courses were slightly modified for the calculation of the out components that contributed to the cruciality measures in this research. Some universities in the MinnState system listed prerequisites using only the lowest required course. Other universities would list the lowest required course that could satisfy a prerequisite and also higher-level courses that could also satisfy the prerequisite. For example, if Precalculus was a prerequisite for a chemistry course then Calculus 1 should also satisfy that prerequisite if Precalculus is a prerequisite for Calculus 1. In cases like this, some universities would only list Precalculus as a prerequisite in their online course catalog and others would list both Precalculus and Calculus 1. A programming command in Mathematica was written so that only the lowest level mathematics courses that satisfied the prerequisites were used in the calculations for cruciality. This adjustment kept the mathematics courses from having artificially inflated out-component sizes. The courses from other departments that had large cruciality measures were also inspected for similar artificially high out-component numbers, and it was determined that no adjustments were needed.

For the universities in the MinnState system, the courses with the highest cruciality measure on each campus were from the mathematics department. In six of the seven universities, the course with the highest cruciality measure was College Algebra. At Metropolitan State University, the course with the highest cruciality measure was MATH 102: Mathematics of Sustainability which can serve as a prerequisite to College

Algebra. Additionally, College Algebra had the third highest cruciality measure for Metropolitan State University (see Table 6). College Algebra was the only course to appear on all seven lists. The courses with the highest cruciality measures at each university had measures ranging from 152 to 410. Since the longest geodesic path in the MinnState system included only ten courses (see Tables 4-5), it is clear that the large cruciality scores came mostly from the size of each course's out-component. With the previously mentioned adjustment to mathematics courses that serve as course prerequisites, the average number of mathematics courses in the top ten cruciality measures across the MinnState system was 4.7. Without the adjustment to the way prerequisites were written, the average number of mathematics courses in the top ten cruciality measures would have been 6.4. Winona State University had the largest number of mathematics courses in its top ten cruciality measures with seven and Bemidji State University had the lowest with only two. Other departments that had courses show

Table 6

College-Level Courses with Highest Cruciality in the Minnesota State Universities

Metropolitan		Mankato		St. Cloud	
MATH 102	282	MATH 112	410	MATH 112	317
WRIT 131	271	MATH 115	313	MATH 115	273
MATH 115	218	MATH 121	292	MATH 221	178
STAT 201	154	MATH 113	292	PHYS 234	130
WRIT 121	99	MATH 122	134	CHEM 160	125
MATH 120	99	PHYS 221	117	CHEM 210	119
WRIT 132	98	CHEM 104	85	MATH 113	107
PSYCH 100	72	PHYS 222	64	MATH 222	99
MATH 215	72	CHEM 201	64	MATH 111	99
ICS 140	69	MATH 321	55	GENG 102	92

Note: The number to the right of each course is its cruciality measure. Some of the mathematics course names can be found in Appendix H. This information is continued in Table 7.

up in the top ten cruciality scores included accounting, art, biology, chemistry, English, economics, finance, physics, psychology, statistics, and writing (see Tables 6-7).

Table 7

College-Level Courses with the Highest Cruciality in the Minnesota State Universities

Southwest		Moorhead		Winona		Bemidji	
MATH 110	153	MATH 127	152	MATH 115	248	MATH 1170	142
MATH 135	80	ART 125	60	MATH 120	239	CHEM 2211	61
ACCT 211	42	ART 101	57	MATH 212	158	CHEM 1111	61
CHEM 231	39	MATH 261	54	ENG 111	138	BIOL 1400	59
FIN 230	32	MATH 227	54	STAT 100	112	ACCT 2101	56
MATH 060	31	MATH 234	49	STAT 110	110	MATH 2471	54
COMP 165	31	PSY 113	45	MATH 110	99	CHEM 2212	51
ECON 201	30	MATH 262	42	MATH 112	82	ECON 2000	50
PSYC 101	29	ACCT 230	36	MATH 117	80	ED 3110	49
MUS 300	29	ACCT 231	32	MATH 140	79	CHEM 1112	49

Note: The number to the right of each course is its cruciality measure. Some of the mathematics course names can be found in Appendix H. This information is a continuation of Table 6.

Hypothesis 4

The fourth hypothesis for this research was that the connectedness of undergraduate curricula in the MinnState university system would be inversely correlated with institution size. A review of the average out-degree for each curriculum prerequisite network in the MinnState university system (see Table 8) closely follows the results from Table 1 and Table 2. The average out-degree for all courses at Minnesota State University, Mankato, St. Cloud State University, Southwest Minnesota State University, and Minnesota State University, Moorhead are very similar (see Table 8). These four universities also had similar numbers of isolated courses and percentages of linked courses in their networks (see Table 1). Additionally, Table 2 showed that these four universities had similar percentages of linked and total courses in their largest components. The numbers and sizes of weakly connected components were not consistent across all four of these universities, but the pairs of universities in the Large and Medium Carnegie classifications did have similar numbers (see Table 2). These similarities in Tables 1, Table 2, and Table 8 also hold for the other three Universities in the MinnState system. Winona State University and Bemidji State University are both have a Carnegie classification of Master's Granting Universities: Small while Metropolitan State University is classified as a Doctorate/Professional degree-granting institution. These three institutions have similar average out-degrees across all of their courses (see Table 8), similar numbers of isolated courses and percentages of linked courses (see Table 1), and percentages of linked and total courses in their largest component. The size and numbers of the weakly connected component in these three networks were not very similar (see Table 2).

Table 8

Average Out-Degree in the Curriculum Prerequisite Networks for MinnState Universities

Carnegie University Classification	Minnesota State University	Average Out-Degree for All Courses	Average Out-Degree for Linked courses
Doctoral/Professional	Metropolitan	0.65	3.76
Master's: Large	Mankato	0.48	2.60
	St. Cloud	0.45	2.68
Master's: Medium	Southwest	0.44	2.82
	Moorhead	0.44	2.27
Maser's: Small	Winona	0.83	2.96
	Bemidji	0.60	2.51

Chapter V – Discussion

Summary of Findings

This research sought to use curriculum prerequisite networks to analyze the similarities and differences of the curriculum structures for the seven universities in the Minnesota State (MinnState) university system. The prerequisite information for each course at each university was taken from the university's publicly available online course catalog. The data collection for this research proved to be time-consuming and difficult since the presentation, availability, and accuracy of course prerequisites varied from one university to the next.

Some universities only listed the lowest level course that would satisfy a prerequisite, while others listed the lowest level course along with more advanced courses as alternatives. For example, assume that College Algebra is a prerequisite for both Precalculus and General Physics, but no lower-level mathematics course would satisfy the prerequisites for these courses. Some universities would list only College Algebra as a prerequisite for General Physics while others would list the option of satisfying the prerequisite with either College Algebra or Precalculus. These *higher prerequisites* are nearly the opposite situation of the *redundant prerequisites* defined in chapter three (see Figure 4). An example of a redundant prerequisite is where College Algebra is a prerequisite for Precalculus and Precalculus is the lowest-level mathematics course that satisfies the prerequisite for General Physics. There would be no need to list College Algebra as a prerequisite to General Physics since the prerequisite to Precalculus already requires that knowledge base. If it were listed as a prerequisite for General Physics, College Algebra would be a redundant prerequisite. The problems of higher-prerequisites and redundant prerequisites were both partially addressed with adjustments to prerequisite course listings.

In addition to issues of redundant and higher prerequisites, there were also problems caused by the variety of methods used to list corequisites in the online course catalogs. Some corequisites were listed as such, some had each course listed as a prerequisite for the other, and some pairings only had statements in the text of the course descriptions that indicated concurrent enrollment was required. These variations were not unique to particular universities. A single university could have all three of these variations. As a final variation on corequisites, some courses listed corequisites that only applied to specific majors or programs, and all other students could take the course as a stand-alone. Great efforts were made for the sake of the current research to identify all of these corequisite courses and list them as a single node in the curriculum prerequisite networks.

The ease with which the prerequisite data could be found and retrieved from each university's public website also varied. In particular, one university had a course catalog user interface with a search function that would at times not even recognize some of the courses and department prefixes in the curriculum. In these cases, course descriptions and prerequisites could only be accessed by searching through an alphabetical list of all the courses at the university. Additionally, for some universities, it was at times easy to mistakenly link to and review a course catalog from a previous academic year. Using the university website search option for a specific course might bring up the course description and prerequisites from a catalog that could be several years old. It was necessary to carefully inspect the given webpage to determine if the given course information was current. This was particularly problematic with courses that no longer existed in the current online course catalog. These courses that appeared in prerequisites but no longer appeared in the course catalog gave an indication to the accuracy of each universities course catalog. This was the only method in this research that could note the accuracy of prerequisites listed in online course catalogs. The number of non-existent courses that were listed as prerequisites varied from one university to the next. The lowest number of such courses at a university was ten and the highest number was sixty. Since the number of listed courses MinnState universities varied from 1,138 to 3,248 (see Table 1), there was only a small percentage of courses that were found to have inaccurate prerequisites.

The initial rendering of the curriculum prerequisite networks for the universities in the MinnState system showed a few consistent features. Each university had one large weakly connected component that accounted for 22.1%-49.2% of all courses at the university and 60.0%-82.4% of all linked courses (see Table 2). Each curriculum prerequisite network also contained smaller weakly connected components that often only contained courses from one department (see Appendices A-G). The total number of courses, linked courses, isolated courses showed mostly consistent patterns when the universities were grouped by their Carnegie classifications (see Table 1).

Hypotheses

The first hypothesis of the current study predicted that lower-level courses would exhibit a higher variance in being labeled a prerequisite than higher level mathematics courses. The results from the out-degrees of similar mathematics course listings across the MinnState system supported this hypothesis. The lower-level mathematics courses varied drastically in the number of courses that listed them as prerequisites. The variation in the upper-level courses was also quite variable between campuses, but the number of courses that listed the higher-level courses as prerequisites was much lower (see Table 3).

The second hypothesis of the current research predicted that the longest prerequisite paths through the curricula would contain mathematics prerequisites. The results confirmed that mathematics courses were present in a large majority of the longest geodesic paths across the MinnState university system. However, mathematics prerequisites did not exist in all such paths. An interesting pattern to the data was that the percentage of geodesic paths that contained mathematics courses grew as the path lengths increased (see Tables 4-5). Only two universities saw a drop in the percentage of geodesic paths that contained mathematics courses for the longest paths.

The third hypothesis of the current research predicted that mathematics departments would have consistently high centrality in the network analysis. The results partially supported this hypothesis. The construction of the departmental prerequisite networks for each of the universities in the MinnState system showed varying results. The mathematics department was often in the largest cluster of grouped departments, but that cluster did not always appear to be central to the entire network. Some networks had more than one cluster that was central to the network (see Figures 17-18), some had no apparent centralized cluster of departments (see Figures 13, 16, and 19), and some had the cluster that contained the mathematics department as the central cluster (see Figures 14-15).

However, the cruciality measures of courses did appear to partially confirm the third hypothesis. At each university, there were several mathematics courses included in the list of the courses with the highest cruciality measures. Additionally, College Algebra had the highest cruciality measure at each university except for one. At that university, the top cruciality measure was a mathematics course that can serve as a prerequisite for College Algebra, and College Algebra had the third highest cruciality score.

The fourth hypothesis of this research predicted the interconnectedness of undergraduate curricula will be inversely correlated with institutional size. The results failed to support this hypothesis. The average out-degree of all and linked courses (see Table 8), the percentage of linked courses (see Table 1), and the percentage of all and linked courses in the largest weakly connected component (see Table 2) indicated that the universities could be bifurcated into two groups. One group contained the universities classified as Master's Granting Institutions of large and medium sizes and the other group contained the Master's Granting Institutions of small size and the only Doctoral/Professional Granting Institution in the MinnState university system. This may indicate that interconnectedness of a university's curriculum may have more of a relation with a university's mission than with a university's size.

Implications

The work of this research and the previous work upon which it rests demonstrates that organizing curricula into a network representation can provide new and informative measures regarding course interactions and curriculum structure. The apparent confirmation for the first hypothesis indicates that the mathematics courses listed as prerequisites for some similar courses across the Minnesota State (MinnState) university system may be inconsistent. For example, since Metropolitan State University has sixty-five courses with College Algebra as a prerequisite and Winona State University only has three (see Table 3), there may be some prerequisites inconsistencies. These inconsistencies would appear when some of the indicated sixty-five courses at Metropolitan State University had equivalent courses at Winona State University. Similar but less extreme variances existed with several other mathematics courses and pairs of universities (see Table 3).

This research also suggested that mathematics courses appear in the majority of longer prerequisite paths at each university in the MinnState system (see Tables 4-5). In support of hypothesis three, this research showed that College Algebra is the most common course to appear in a prerequisite path leading up to a target course (see Tables 6-7). This implies that success in College Algebra plays a significant role in students' ability to progress through the curriculum and graduate on time. It is questionable if it is possible for a single course to serve all of these target courses. Additionally, the department networks developed to investigate hypothesis three revealed that mathematics departments are often closely grouped with departments that are different from traditional science, technology, and engineering (i.e., STEM) courses. It is possible that the implication of these departmental groupings and the over-dependence on the College Algebra course indicates a need for that more service courses in the mathematics departments.

A final broad implication of this research is that there exists a largely untapped wealth of information in university course catalogs. Simply listing the out-components for courses could be beneficial for student advising and faculty course planning. It is not unknown to many faculty members that College Algebra is a course that will satisfy many prerequisites at a given university. However, it may surprise faculty and students how far across the curriculum the out-components of College Algebra and other mathematics courses actually reach. Listing the out-components for courses would show students the prerequisite paths that open up for them upon completion of a course as opposed to choosing courses to satisfy the prerequisite path to a target. Similarly, listing the out-components of courses would help faculty determine the audience for a course and help in structuring course content.

Strengths and Limitations

The major strength of this research is that the analysis was performed across all the universities in one of the nation's largest state-wide systems. Some studies have limited their analysis to the curricula of a small number of colleges or universities (Aldrich, 2015; Knorn et al. 2019, Lightfoot 2010; Molontay, 2020; Slim et al. 2014; Varagnolo et al. 2020; Wigdahl et al. 2014). Others have considered a larger number of schools, but the similarities between the institutions have been based on similar programing and not a shared board of trustees (Heileman et al., 2019; Komenda et al., 2015). The consistent collection and analysis of curriculum data from universities in a single system allows for a unique look at how local academic decisions can lead to structural differences.

The focus on how mathematics courses and departments fit into a university's curriculum is also a strength of this research. Much of the research using curriculum analytics reviews particular programs in a university's curriculum but not the curriculum as a whole (Aldrich, 2015; Heileman et al., 2019; Knorn et al. 2019, Lightfoot 2010; Molontay, 2020; Slim et al. 2014; Varagnolo et al. 2020; Whigdahl et al. 2014). Since mathematics courses are often taken by students majoring in other departments

(American Mathematical Society, 1999; Mathematical Association of America, 2015; Herriott and Dunbar 2009), mathematics courses and departments play a central role in each university's curriculum as a whole.

Finally, the use of network analytics to define the structure of a university's curriculum is a relatively new process. The current makeup of higher education is one where faculty and administration are increasingly relied upon to make data-driven decisions. The use of curricular network analytics could prove to be a valuable tool used to quantifying data relates to course decisions and curriculum structure.

The largest limitation to this research is the accuracy of and adjustments made to the listed prerequisites for each course. The prerequisite data for each university was taken from the online course catalogs. As stated previously, each university had some courses listed as prerequisites, but those courses no longer existed in the current course catalog. It is also possible that course catalogs had courses listed even though departments had determined that those courses would no longer be offered. This research made no effort to confirm which courses were perennially offered at each institution. Additionally, it is possible, and indeed likely, that some course prerequisites that were found to contain non-existent courses had been updated by the appropriate departments, but those updates were not added to the current course catalog. When it was possible to find these courses with prerequisite errors on the MinnState e-Services course registration website, the appropriate prerequisites were substituted into the data. However, the public information for the e-Services site typically only has three semesters of scheduling available for review. If a course was not offered in that window of time, the current and correct prerequisite courses were not known to this research.

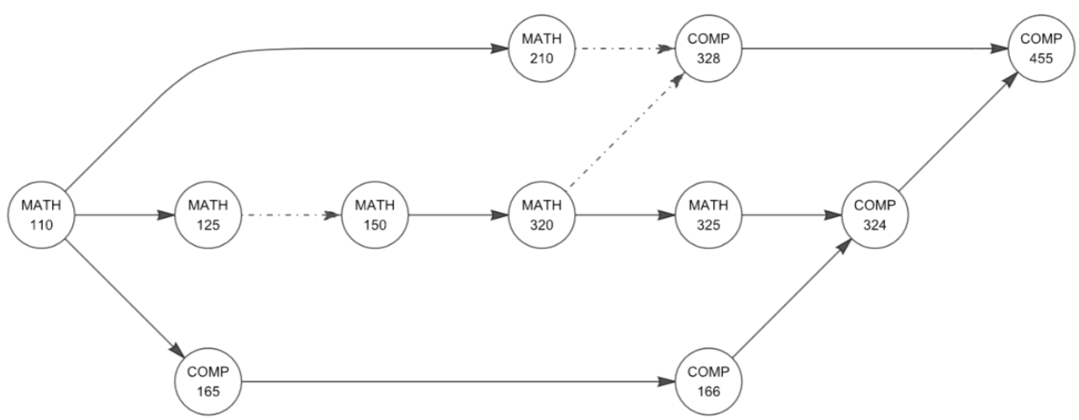
An additional limitation of this research is the difficulty in adapting network curriculum analytics found in the literature to weighted prerequisite connections. The edge weighting for the curriculum prerequisite networks was based on the statement of the prerequisite. If there was an “Or” in the prerequisite, then an edge was listed with a weight less than one (see Figure 1). These “Or” listings make it difficult to find redundant prerequisites (see Figure 4), properly scale out-components (see Tables 6-7), and accurately find geodesic paths (see Tables 4-5).

As stated in chapter three, an adjustment was made to remove redundant prerequisites that strictly came from “And” statements, but the redundant prerequisites that came from a path with “Or” statements were not adjusted. Similarly, as stated in chapter four, several mathematics courses initially had inflated out-component calculations due to “Or” prerequisites statements. An adjustment was made to these prerequisite statements. This adjustment led to more accurately listing the average number of mathematics courses in the top ten cruciality scores as 4.7 as opposed to the pre-adjustment average of 6.4 (see Tables 6 -7) . However, the out-component calculation still does not account for edge (prerequisite) weighting. For example, assume either MATH 105 or MATH 107 satisfied the prerequisite for MATH 165, and assume that MATH 165 is a prerequisite for CHEM 475 (see Figure 1). The edge from MATH 105 to MATH 165 adds one-half to the out-degree of MATH 105 but the same edge adds one to the number of courses in the out-component of MATH 105. Similarly, CHEM 475 would add another full course to the out component of MATH 105.

The number of courses in a path was used to define the geodesic and longest geodesic paths, but the edge (prerequisite) weighting was also not involved in the calculation. Figure 20 shows prerequisite paths to COMP 455 in the Southwest State University curriculum prerequisite network. The geodesic path from MATH 110 to COMP 455 would be the one containing MATH 110, MATH 210, and COMP 455. However, as indicated by the solid edges, the path containing MATH 110, COMP 165, COMP 166, COMP 324, and COMP 455 contains exclusively required courses. Similarly, since COMP 324 is a required prerequisite for COMP 455, the path containing MATH 150, MATH 320, MATH 325, COMP 324, and COMP 455 also exclusively contains required courses. Since there is an unseen alternate option to MATH 125 for MATH 150, it's unclear what would be the shortest way to finish this path. Regardless, both of these required paths would be longer than what this research recorded as the geodesic path from MATH 110 to COMP 455.

Figure 20

A Prerequisite Path from Southwest Minnesota State University



A final limitation to this research is that student enrollment trends and success data were not used in this research. It is possible that a course with no prerequisites that also does not serve as a prerequisite for any other course still has high enrollments and serves a central purpose to the curriculum. Alternatively, it is possible that a course serves as a prerequisite to many courses and has a large number of courses in its out-component, but the course has low enrollments. This could possibly occur in a low-level mathematics course where many students at the university are initially placed at a level above the course due to testing or previous course work. It could also be the case that a course central to the curriculum that has high student success rates would be less of a concern than a course less central and with lower success rates.

Recommendations for Further Research

An extension of this research would be to define the meaning of the “accepted to” prerequisites and include this information in the curriculum prerequisite networks. The “accepted to” prerequisites are the ones that state a student must have been accepted to a program or major before enrollment in a target course. For example, many of the prerequisite networks in the MinnState university system included weakly connected components that only contained courses from a single department (see Appendices A-G) and many isolated courses (see Table 1). Additional linkages could be defined for these components and courses if there are exist prerequisites of a declared major or acceptance to a program and that declaration or acceptance required specific course work. How this would change the centrality and importance of the mathematics departments across university curricula is unknown.

Another extension of this research would be to incorporate student success data or enrollment information into the network analysis in an effort to better understand how students progress through the curriculum. Among others, the work of Molontay et al. (2020), Saltzman and Roeder (2012), Slim et al. (2014), and Wigdahl et al. (2014) gives practical and theoretical examples of how this may be accomplished. Additionally, the comparisons of network analytics could also be done while grouping the universities according to factors besides classified size. Network curriculum analytics could show more consistency when the grouping is based on each university's mission. If a university is more likely to enroll older-than-average and returning students then the listed prerequisites for a course may need to be more or less prescriptive than a university that enrolls primarily residential students straight out of high school. As Heileman et al. (2018) noted, prerequisites are likely more necessary when students are less prepared.

A final recommendation for further research would be to extend this analysis of curriculum prerequisite networks to the community colleges in the MinnState system. In particular, does College Algebra play a similarly central role in the curricula of the community colleges? This additional research could be tailored as a comparison between community colleges or a comparison between a community college and typical transfer institutions. The analysis could aid in determining the consistency of prerequisites from two to four-year colleges and possibly help with identifying issues that could arise with the transfer between institutions.

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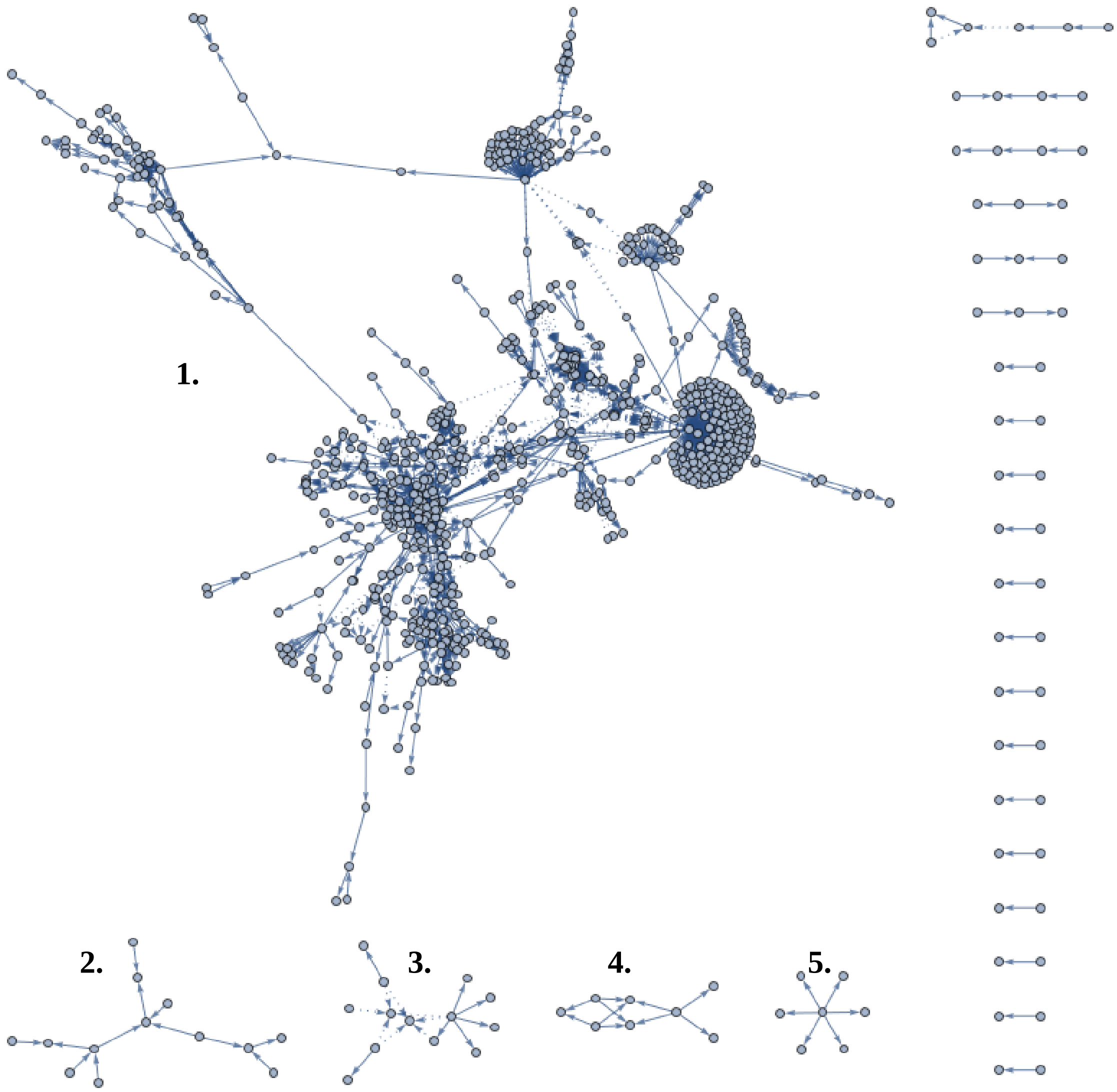
Appendix A

Components for the Metropolitan State University

Curriculum Prerequisite Network



Note: Departments of Courses in the Indicated Components, Ordered by Component Size

1. Accounting, Anthropology, Biology, Business Law, Computer Forensic Sciences, Chemistry, Criminal Justice, Communication, Cybersecurity, Data Science, Dental Hygiene, Decision Sciences, Economics, Education, Entrepreneurship,

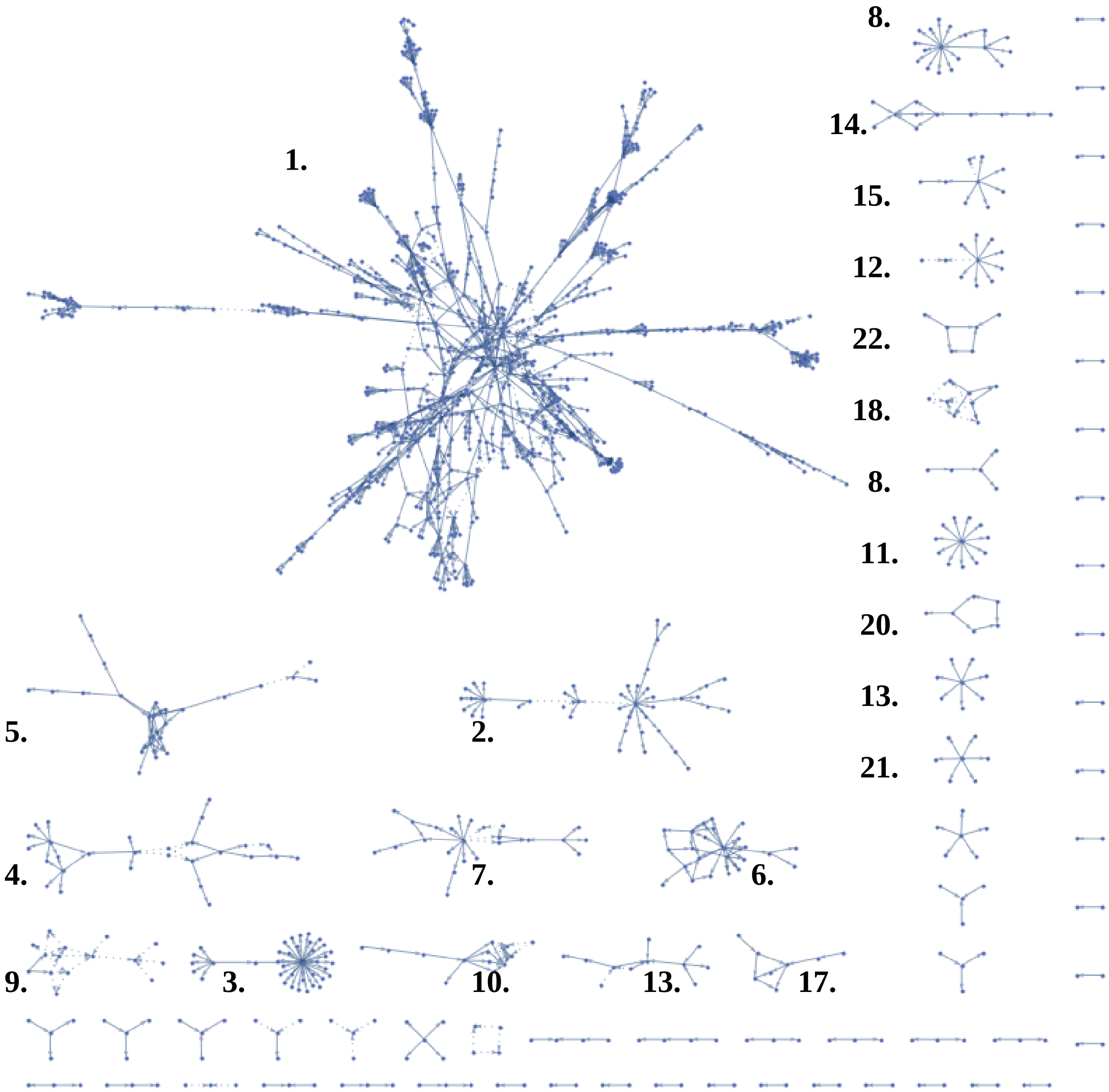
Environmental Science, Ethnic Studies, Finance, Geology, Gender Studies, History, Human Services Administration, Health Science, HSCO, Human Services, Human Services/Family Studies, Humanities, International Business, Information and Computer Sciences, Interdisciplinary Studies, Information Studies, Language Arts Education, Law Enforcement, Linguistics, Literature, Mathematics Education, Mathematics, Media Studies, Management, Management Information Systems, Marketing, Advertising and Purchasing, Natural Science, Nursing, Philosophy, Physics, Political Science, Psychology, Risk Management and Insurance, Science Education, Screenwriting, Sociology, Social Work, Special Education, Social Science, Social Studies Education, Statistics, Technical Communication/Interaction Design, Theater, Writing

2. Arts
3. Nursing
4. Criminal Justice, Human Services/Alcohol & Drug Counseling, Human Services
5. Human Resource Management

Appendix B

Components for the Minnesota State University, Mankato

Curriculum Prerequisite Network



Note: Departments of Courses in the Indicated Components, Ordered by Component Size

1. Accounting, Automotive Engineering Technology, Anthropology, Astronomy, Aviation, Biology, Chemistry, Computer Information Science, Civil Engineering, Construction Management, Computer Science, Dental Hygiene, Economics,

Electrical Engineering, Electrical Engineering Technology, English, Integrated Engineering, Ethnic Studies, Family Consumer Science, Finance, Geography, Geology, Health Science, Human Performance, International Business, Mass Media, Mathematics, Mechanical Engineering, Manufacturing Engineering Technology, Management, Marketing, Music General, Nursing, Physics, Psychology, Social Work, Statistics

2. Theatre Arts
3. French
4. Art
5. Art
6. Recreation, Parks and Leisure Services
7. Criminal Justice, Corrections, Health Science, Human Performance, Sociology
8. Spanish
9. English, Film
10. German
11. Political Science
12. Health Science
13. Dance
14. Health Science
15. Geography
16. Business Law
17. Integrated Engineering
18. Social Work

19. Political Science

20. Human Performance

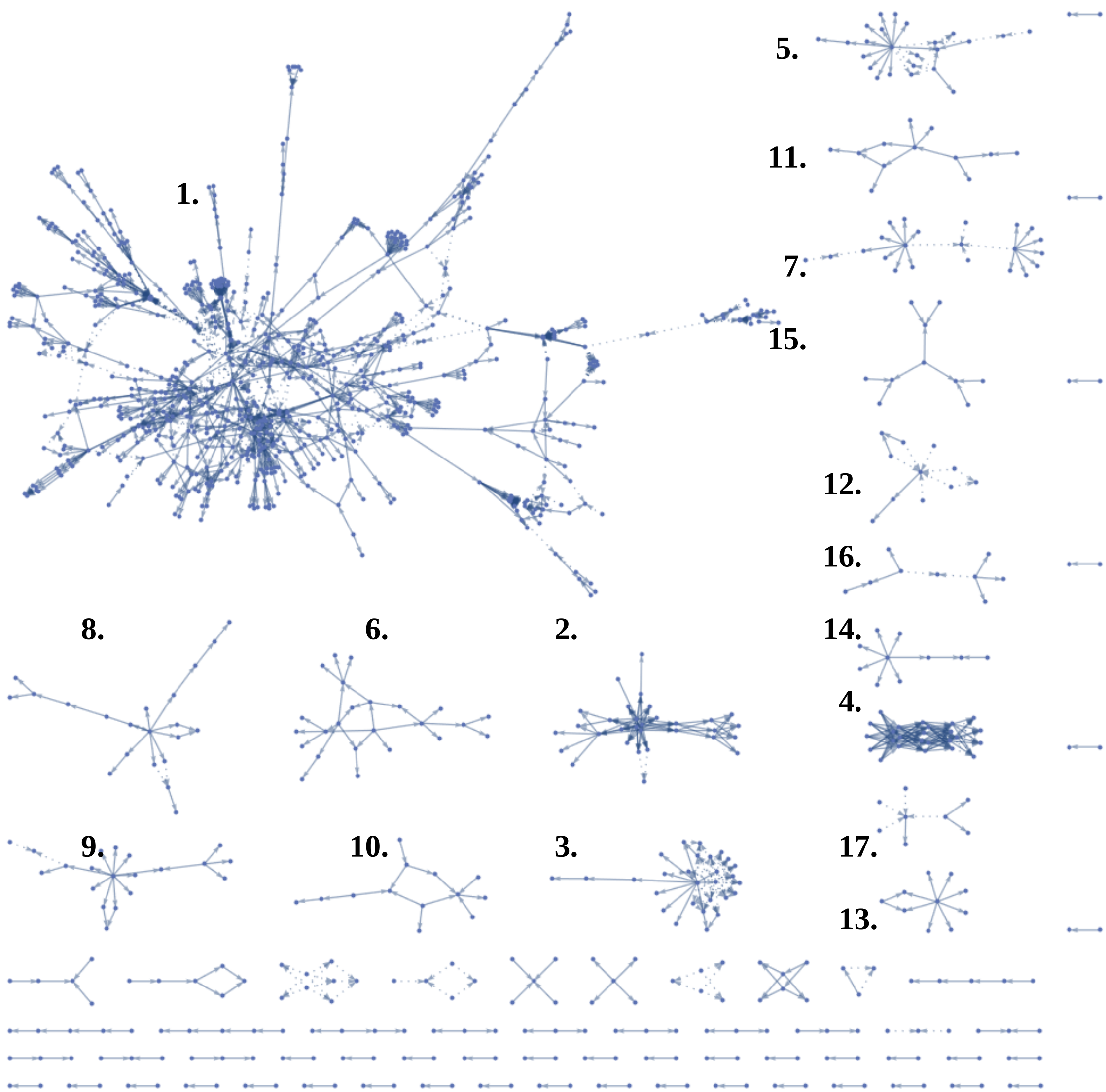
21. Ethnic Studies

22. Nursing

Appendix C

Components for the St. Cloud State University

Curriculum Prerequisite Network



Note: Department of Courses in the Indicated Components, Ordered by Component Size

1. Accounting, Atmospheric & Hydrologic Sciences, Astronomy, Biological Sciences, Biochemistry& Molecular Biology, Child and Family Studies, Chemistry, Communication Studies, Computer Networking and Applications, Community

Psychology, Computer Science, Electrical and Computer Engineering, Economics, Education, Engineering Management, English, Engineering Science, Environmental Engineering, Ethnic Studies, Environmental and Technological Studies, Finance, Insurance and Real Estate, General Engineering, Geography, Gender and Women's Studies, Health, Human Relations and Multicultural Ed, Information Media, Information Systems, Mathematics, MATS, Management, Marketing and Business Law, Medical Laboratory Sciences, Mechanical and Manufacturing Engineering, Mathematics and Statistics, Music Education, Physical Education Sports Science, Philosophy, Physics, Psychology, Science, Software Engineering, Sociology, School of the Arts, Special Education, Social Studies, Statistics, STEM, Social Work

2. Art
3. Criminal Justice Studies
4. Nursing
5. Anthropology, Sociology
6. Mass Communications
7. Political Science
8. Music Education, Music Musicianship, Music Performance
9. Communication Sciences and Disorders, English, German, Languages and Cultures, Spanish
10. Health Education and Physical Education, Physical Education Sports Science

11. Physical Education Sports Science, Recreation

12. History

13. Hospitality and Tourism

14. Global Studies, Political Science

15. Special Education

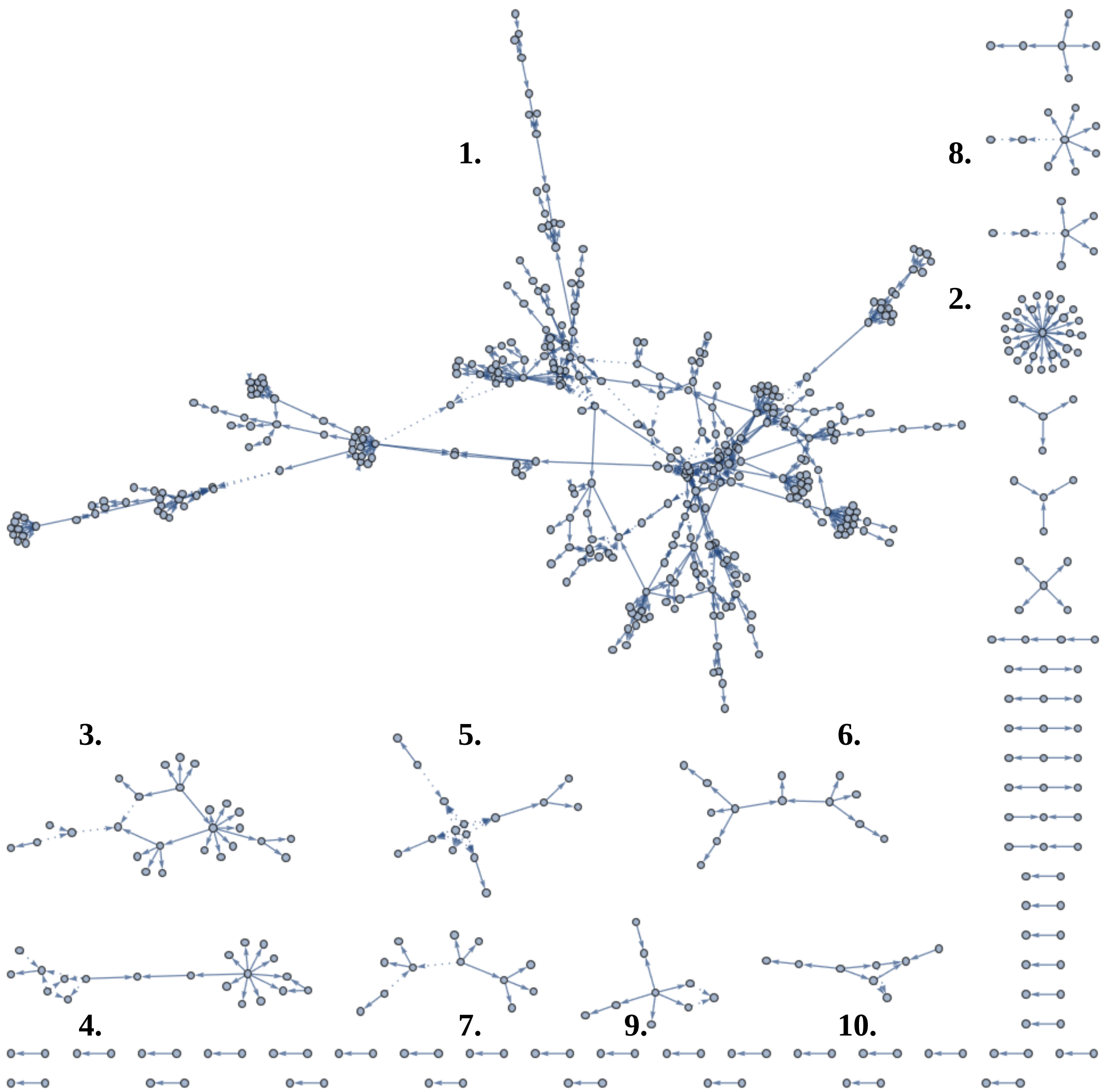
16. Anthropology

17. Theatre

Appendix D

Components for the Southwest Minnesota State University

Curriculum Prerequisite Network



Note: Departments of Courses in the Indicated Components, Ordered by Component Size

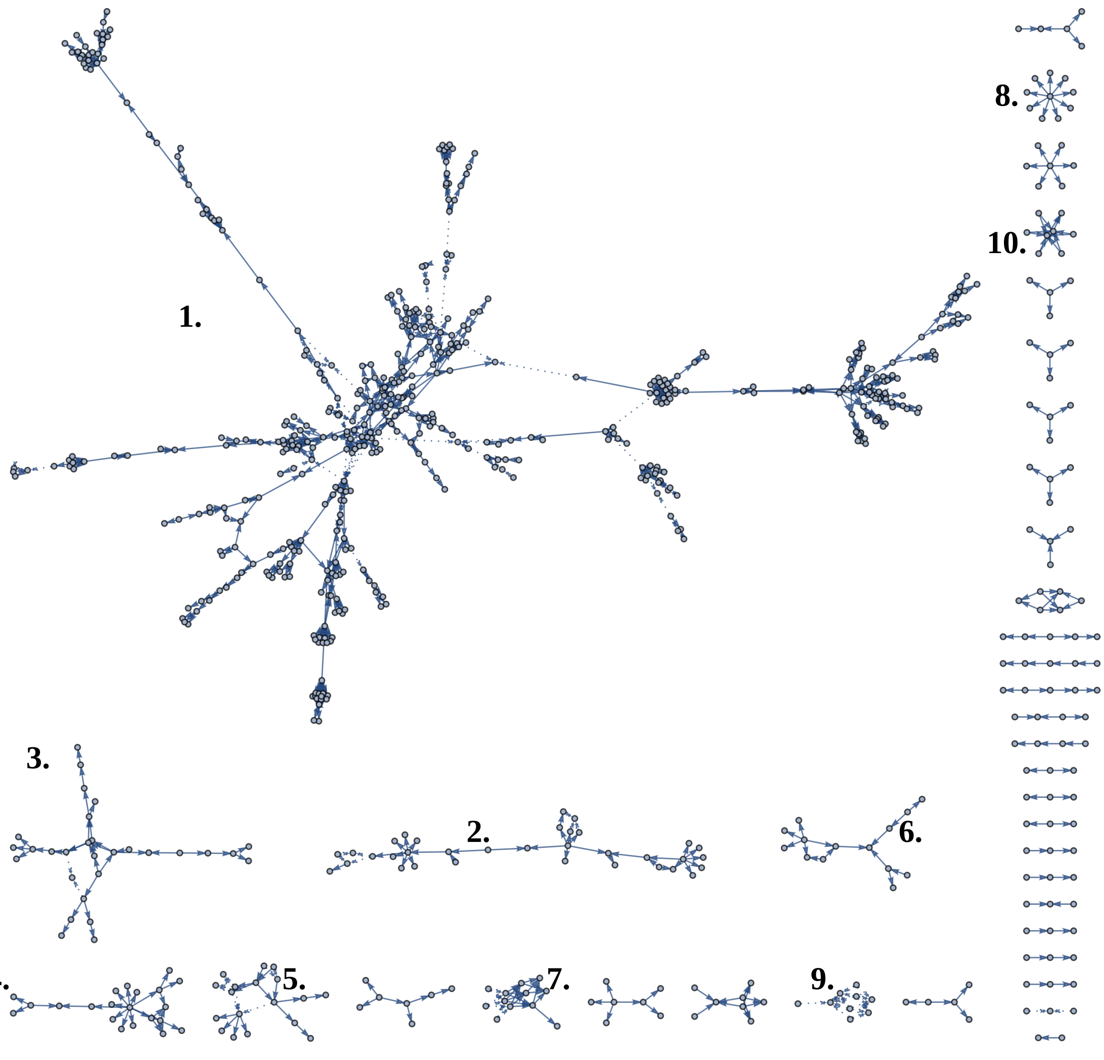
1. Accounting, Agribusiness Management, Agronomy, Agricultural Solutions, Art, Biology, Chemistry, Computer Science, Culinology, Data Science, Economics, Early Childhood Special Education, Education, Environmental Science, Exercise Science,

Finance, Health, Hospitality, Justice Administration, Mathematics, Management, Marketing, Physical Education, Physics, Political Science, Psychology, Sociology, Special Education, Social Work

2. Music
3. Music
4. English: American Language, Marketing, Philosophy
5. Art
6. Theatre Arts
7. History, Spanish, Teaching English as a Second Language
8. Justice Administration, Political Science
9. Communication Studies
10. Education, Spanish

Appendix E

Components for the Minnesota State University, Moorhead Curriculum Prerequisite Network



Note: Departments for Courses in the Indicated Components, Ordered by Component Size

1. Accounting, Anthropology, Art, Astronomy, Biochemistry and Biotechnology, Biology, Chemistry, Criminal Justice, Construction Management, Computer Science and Information Systems, Economics, Education, Elementary and Early Childhood

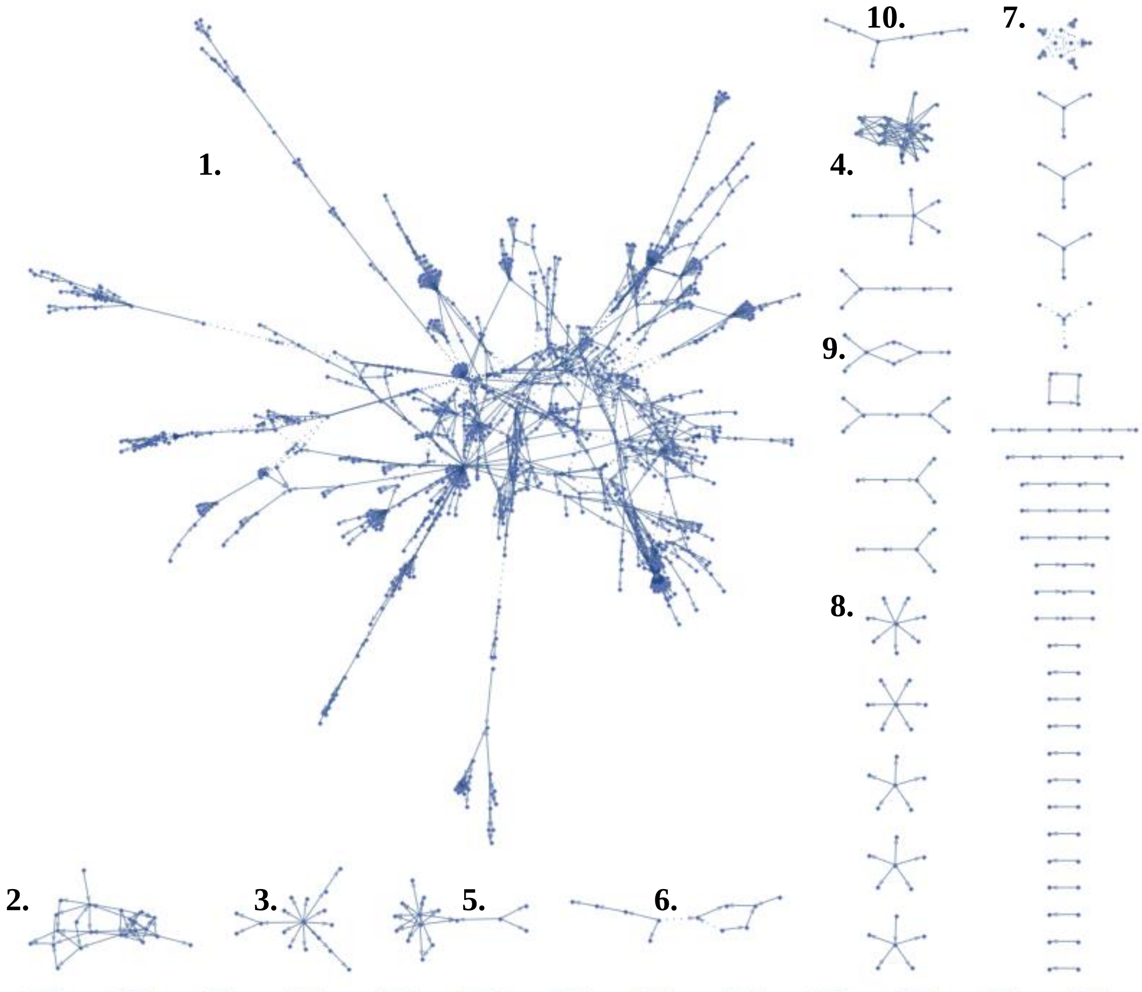
Education, English, Finance, Geoscience, Graphic and Interactive Design, Honors, Health Services Administration, Mathematics, Management, Marketing, Operations Management, Paralegal, Philosophy, Physics, Project Management, Political Science, Psychology, Speech, Language & Hearing Science, Sociology, Special Education, School of Teaching and Learning, Social Work, Women's Studies

2. Animation, Film Studies, Photography
3. Music
4. Entertainment Industry Technology, Paralegal
5. Spanish
6. Speech Language & Hearing Science
7. Nursing
8. Anthropology, Women's Studies
9. Philosophy
10. English, History, Physical Science, Sustainability, Women's Studies

Appendix F

Components for the Winona State University

Curriculum Prerequisite Network



Note: Departments of Courses in the Indicated Components, Ordered by Component Size

1. Accounting, Art & Design, Bilingual/Bicultural Education, Biology, Business Administration, Chemistry, Composite Materials Engineering, Communication Studies, Clinical Practice Education Studies, Computer Science, CSED, Data Science, Economics, Educational Foundations, Education, English, Ethnic Studies, Film, Finance, Geoscience, Global Studies, Health Administration, Health Exercise

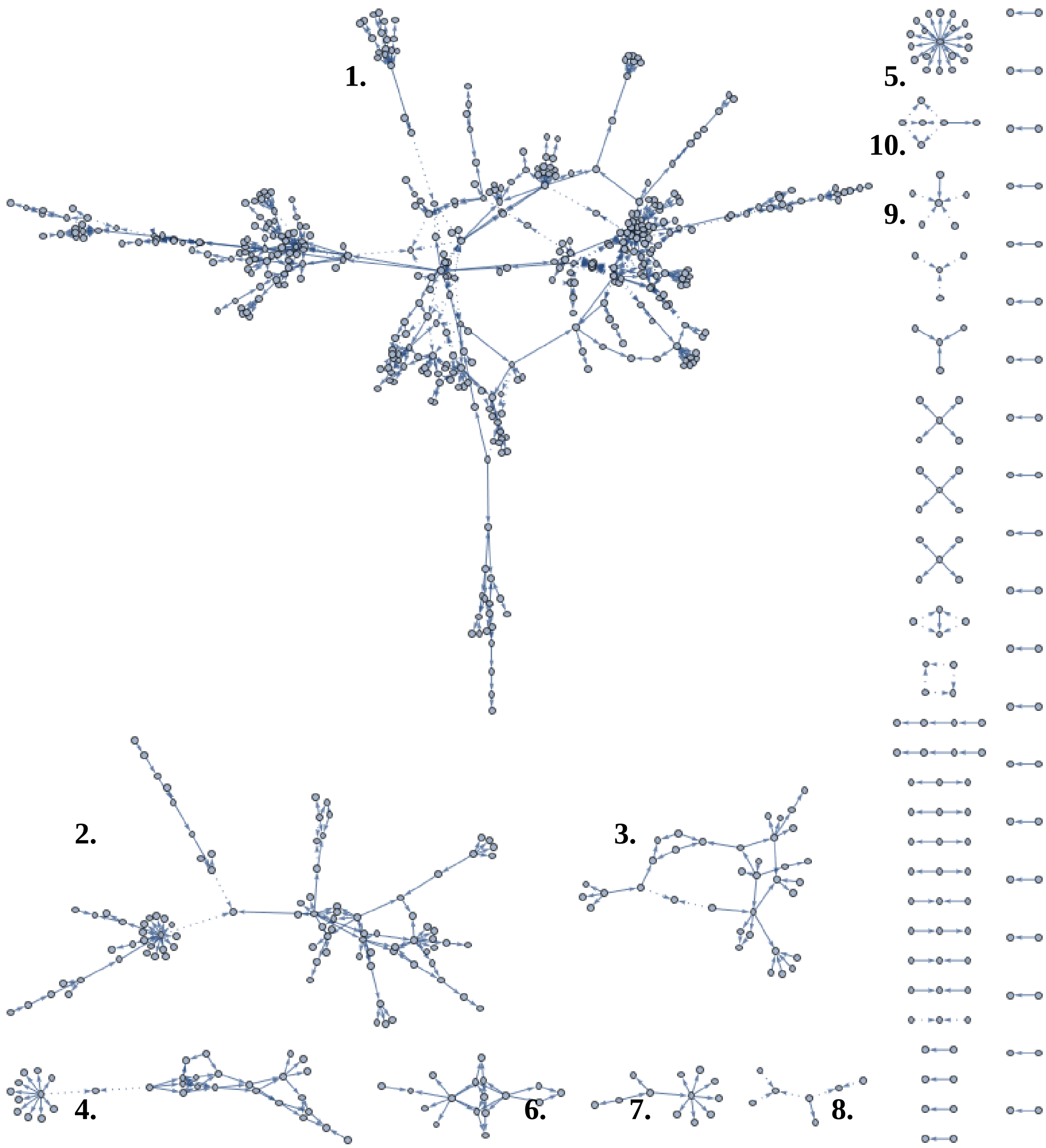
Rehabilitative Sciences, History, Healthcare Leadership and Administration, Individualized Studies, Legal Studies, Mathematics, Mass Communication, Management, Management Information Systems, Marketing, Music, Nursing, Physical Education and Sport Science, Physics, Political Science, Psychology, Sociology, Social Work, Spanish, Special Education, Statistics, Sustainability, Theatre & Dance, Women's Gender and Sexuality Studies, World Language Education

2. Nursing
3. POL
4. Rochester Education Department
5. Recreation, Tourism & Therapeutic Rec
6. Mass Communication
7. Biology, Chemistry, Geoscience, Physics
8. Music, Theatre & Dance
9. History
10. Child Advocacy Studies

Appendix G

Components for the Bemidji State University

Curriculum Prerequisite Network



Note: Departments of Courses in the Indicated Components, Ordered by Component Size

1. Accounting, Biochemistry, Cellular & Molecular Biology, Biology, Business Administration, Chemistry, Criminal Justice, Computer Science, Economics, English, Environmental Studies, Geography, Geology, Health, Mass Communications, Mathematics, Physical Education, Physics, Political Science, Psychology, Sociology, Social Work, Spanish, Statistics, Technology, Art and Design -Technology
2. Professional Education, Mathematics, ML, Ojibwe, Physical Education, Spanish, Special Education
3. Music
4. Nursing
5. Music
6. Technology, Art and Design - Design
7. Mass Communications
8. History
9. Indigenous Studies
10. Geography

Appendix H

Equivalent Mathematics Courses in the MinnState University System Results are Based on Transferology.com

Course	Metro.	Mankato	St. Cloud	SMSU	Moorhead	Winona	Bemidji
Int. Algebra	098	098	072	060	099	050	0800
College Algebra	115	112	112	110	127	115	1170
Trigonometry	--	113	113	125	143	--	1180
Precalculus	120	115	115	135	142	120	1470
Calculus I	210	121	221	150	261	212	2471
Calculus II	211	122	222	151	262	213	2472
Calculus III	310	223	321	252	323	312	2480
Discrete Math.	215	280	271	210	210	247	2210
Linear Algebra	315	247	312	360	327	242	3310
Differential Equ	350	321	325	350	366	313	2490
Real Analysis	301	417	421	450	361	452	4410
Abstract Algebra	471	345	461	440	476	447	4371

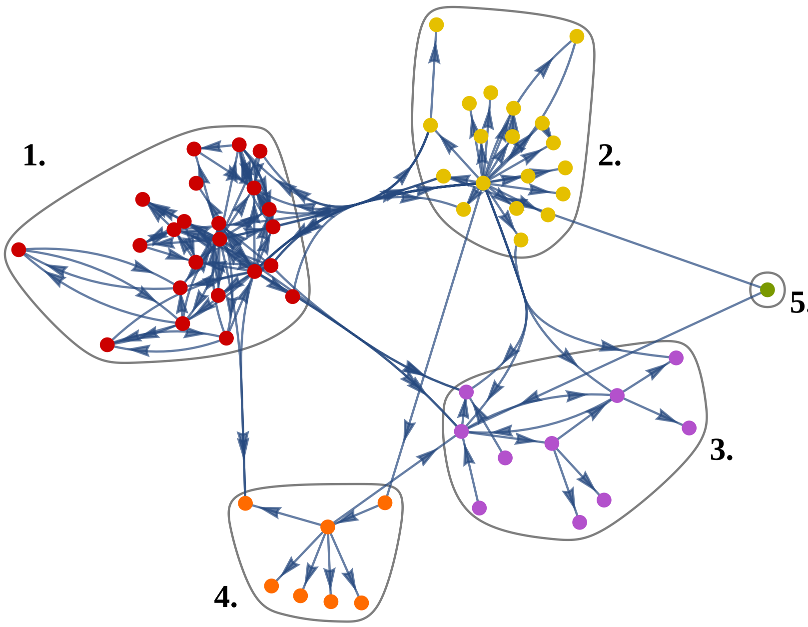
Note: The department name for each course number is MATH. Metro.= Metropolitan State University, SMSU= Southwest Minnesota State University,

Int. Algebra= Intermediate Algebra, Discrete Math.=Discrete Mathematics, Differential Equ.=Differential Equations

Appendix I

The Departmental Curriculum Prerequisite Network for

Metropolitan State University



Note: Departments in the indicated clusters, ordered by cluster size.

1. Accounting, Biology, Chemistry, Computer Forensic Sciences, Cybersecurity, Data Science, Decision Sciences, Economics, Entrepreneurship, Environmental Science, Finance, Geology, Information and Computer Sciences, International Business, Management, Management Information Systems, Mathematics, Natural Science, Nursing, Physics, Risk Management and Insurance, Social Work, Statistics
2. Advertising and Purchasing, Anthropology, Business Law, Communication, Dental Gender Studies, Hygiene, History, Humanities, Information Studies, Interdisciplinary Studies, Literature, Marketing, Media Studies, Political Science, Screenwriting, Social Science, Sociology, Technical Communication/Interaction Design, Theater, Writing

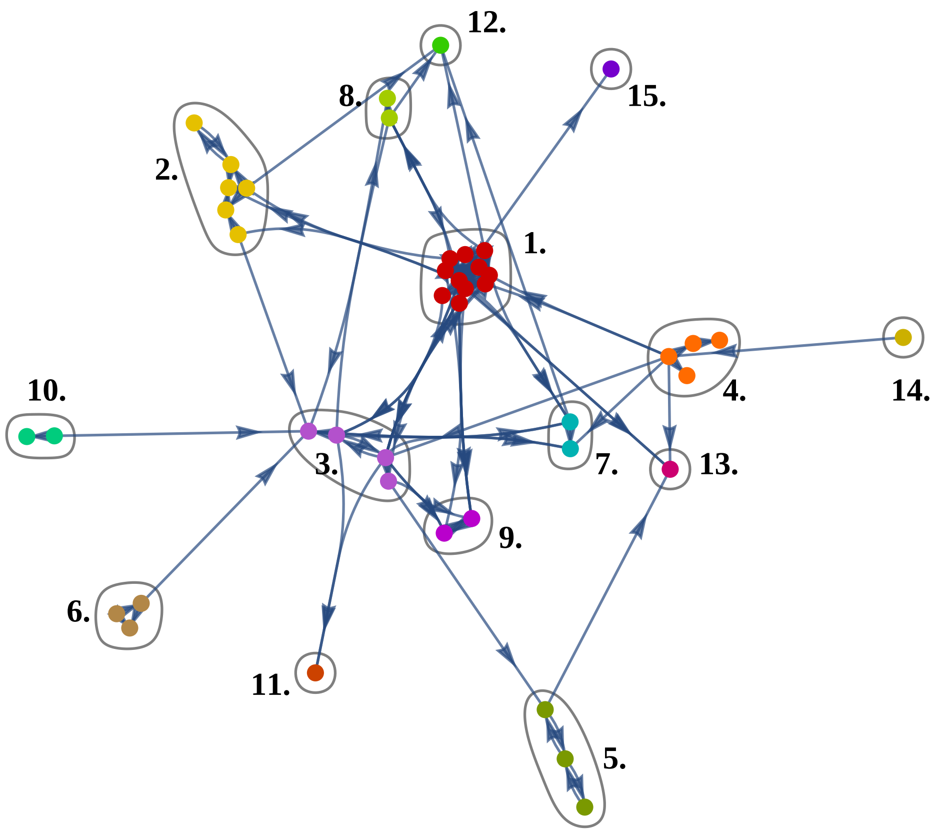
3. Criminal Justice, Ethnic Studies, Health Science, Human Services Corrections, Human Services, Human Services Administration, Human Services/Alcohol & Drug Counseling, Human Services/Family Studies, Law Enforcement, Psychology
4. Education, Language Arts Education, Mathematics Education, Science Education, Special Education, Social Studies Education, Linguistics
5. Philosophy

Isolated Departments: CAS, CC, DKTA, Geography, Human Services and Disability Studies, Human Services Gerontology, Human Service Violence Prevention, Metro Educational Planning, Music, Nonprofit Management, Ojibwe, Public Administration, Reading, Religious Studies, Speech, UMET

Appendix J

The Departmental Curriculum Prerequisite Network for

Minnesota State University, Mankato



Note: Departments in the indicated clusters, ordered by cluster size.

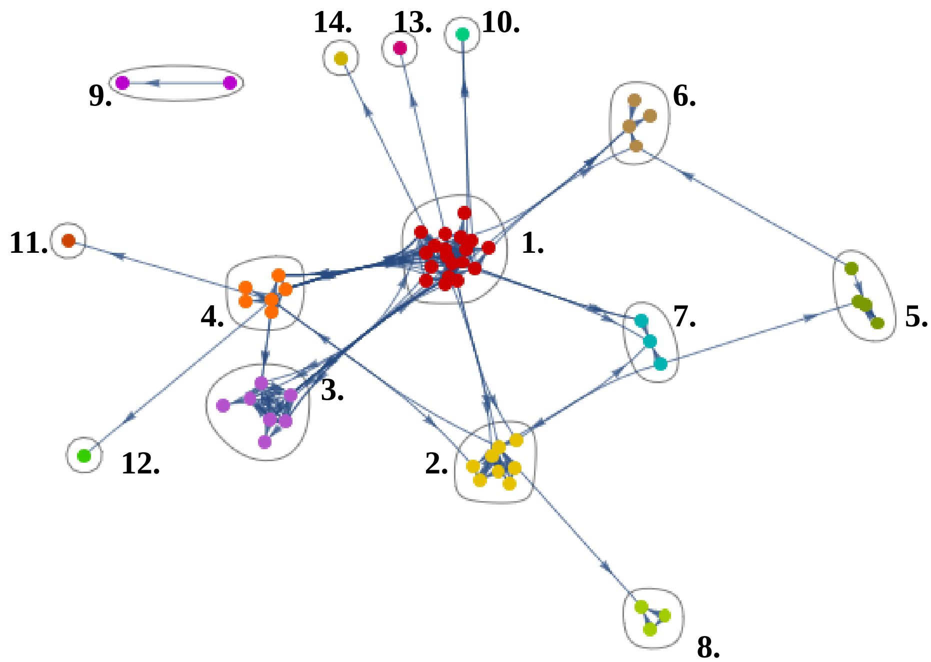
1. Astronomy, Automotive Engineering Technology, Computer Information Science, Computer Science, Construction Management, Electrical Engineering, Electrical Engineering Technology, Manufacturing Engineering Technology, Mathematics, Physics, Statistics
2. Accounting, Economics, Finance, International Business, Management, Marketing

3. Biology, Chemistry, Geology, Human Performance
4. English, Mass Media, Music General, Music Performance
5. Anthropology, Ethnic Studies, Geography
6. Criminal Justice, Corrections, Sociology
7. Family Consumer Science, Health Science
8. Nursing, Psychology
9. Civil Engineering, Mechanical Engineering
10. Educational Studies: K-12 & Secondary Programs, SPST
11. Dental Hygiene
12. Social Work
13. Aviation
14. Film
15. Engineering

Isolated Departments: CAHN Interdisciplinary, Chinese, Communication, Counseling and Student Personnel, Dakota Language, First Year Experience, Gerontology, History, Interdisciplinary Studies, Medical Technology, Museum Studies, Rehabilitation Counseling, Special Education, Urban and Regional Studies, World Languages and Cultures

Appendix K

The Departmental Curriculum Prerequisite Network for St. Cloud State University



Note: Departments in the indicated clusters, ordered by cluster size.

1. Atmospheric & Hydrologic Sciences, Astronomy, Biological Sciences, Chemistry, Computer Networking and Applications, Computer Science, Economics, Engineering Science, Environmental and Technological Studies, Environmental Engineering, Geography, Math Education, Mathematics, MATS, Mechanical and Manufacturing Engineering, Medical Laboratory Sciences, Physics, Software Engineering, Statistics
2. Child and Family Studies, Community Psychology, Education, Information Media, Special Education, Science, Science, Social Studies, Technol, Engineer & Math Education

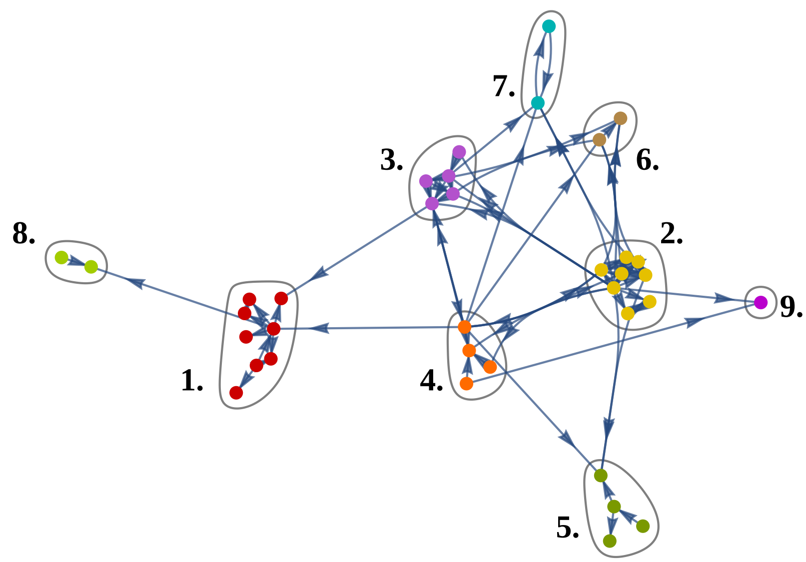
3. Accounting, Business Law, Finance, Insurance and Real Estate, Information Systems, Management, Marketing and Business Law, School of the Arts
4. Communication Studies, Electrical and Computer Engineering, English, General Engineering, Languages and Cultures, Spanish
5. Anthropology, Ethnic Studies, Gender and Women's Studies, Sociology
6. Physical Education Sports Science, Health, Health Education and Physical Education, Recreation
7. Psychology, Social Work, Human Relations and Multicultural Ed
8. Music Education, Music Musicianship, Music Performance
9. Global Studies, Political Science
10. Biochemistry & Molecular Biology
11. Communication Sciences and Disorders
12. German
13. Philosophy
14. Engineering Management

Isolated Departments: British Studies, Construction Management, Community Studies, College Experience, English for Academic Purposes, East Asian Studies, Education Administration, Exercise Science, French, Gerontology, Herberger Business School, Honors, Humanities, Information Assurance, Jewish Studies, Learning Resources and Services, Military Science, Music, Nuclear Medicine Technology, Radiologic Technology, Religious Studies, School of Health and Human Services

Appendix L

The Departmental Curriculum Prerequisite Network for

Southwest Minnesota State University



Note: Departments in the indicated clusters, ordered by cluster size.

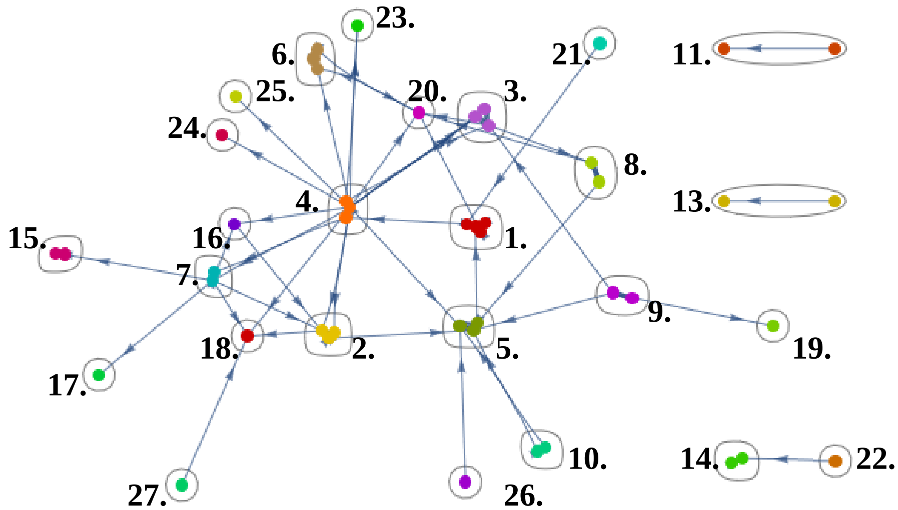
1. Education, Early Childhood Special Education, Health, History, Physical Education, Special Education, Spanish, Teaching ESL
2. Accounting, Agribusiness Management, Finance, Computer Science, Data Science, Economics, Management, Mathematics
3. Biology, Chemistry, Environmental Science, Exercise Science, Physics
4. Social Work, Psychology, Sociology, Political Science
5. English: American Language, Literature, Marketing, Philosophy
6. Agronomy, Agricultural Solutions
7. Culinology, Hospitality
8. Art, Honors Program
9. Justice Administration

Isolated Departments: Agricultural Education, ANSC, Anthropology, Business Law, Geography, Humanities, Interdisciplinary, Indigenous Nations Dakota Studies, Liberal Education Program

Appendix M

The Departmental Curriculum Prerequisite Network for

Minnesota State University, Moorhead



Note: Departments in the indicated clusters, ordered by cluster size.

1. English, Sustainability, History, Physical Science
2. Management, Economics, Marketing
3. Biochemistry and Biotechnology, Biology, Chemistry
4. Computer Science and Information Systems, Mathematics, Physics
5. Women's Studies, Sociology, Social Work
6. Education, Elementary and Early Childhood Education, Special Education
7. Accounting, Construction Management
8. Anthropology, Geoscience
9. Art, Psychology
10. Criminal Justice, Political Science
11. English Language Program, Teaching English as a Foreign

12. Animation, Photography

13. Paralegal, Entertainment Industry Technology

14. Physical Education, Athletic Training

15. Finance

16. Health Services Administration

17. Operations Management

18. Graphic and Interactive Design

19. School of Teaching and Learning

20. Communication

21. Film Studies

22. Astronomy

23. Honors

24. Speech, Language & Hearing Science

25. Philosophy

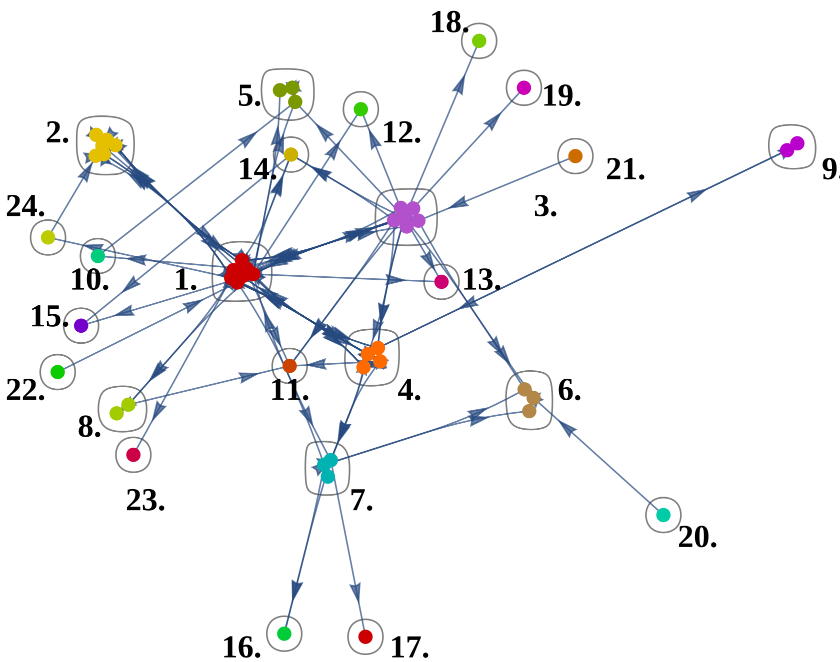
26. Project Management

Isolated Departments: American Multicultural Studies, Athletics, Business, Engineering, Entrepreneurship, Exchange, First Year Experience, Humanities, International Studies, Library, Media Arts, University Studies

Appendix N

The Departmental Curriculum Prerequisite Network for

Winona State University



Note: Departments in the indicated clusters, ordered by cluster size.

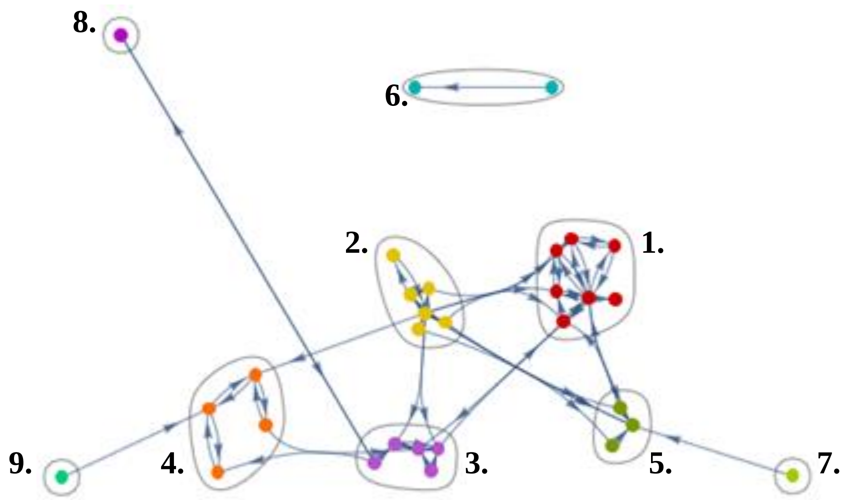
1. Computer Science, Physics, Composite Materials Engineering, Communication Studies, Data Science, Statistics, Mathematics
2. Accounting, Management, Business Administration, Management Information Systems, Economics, Finance
3. English, Mass Communication, Psychology, Global Studies, History, Women's, Gender, and Sexuality Studies
4. Biology, Chemistry, Geoscience, Health Exercise Rehabilitative Sciences
5. Theatre & Dance, Music
6. Bilingual/Bicultural Education, World Language Education, Spanish

7. Education, Educational Foundations, Clinical Practice Education Studies
8. Healthcare Leadership and Administration, Health Administration
9. Physical Education and Sport Science, Clinical Practice in Physical Education
10. Art & Design
11. Nursing
12. Film
13. Political Science
14. Sociology
15. Social Work
16. Special Education
17. Education - Reading
18. Individualized Studies
19. Sustainability
20. Ethnic Studies
21. Japanese
22. Legal Studies
23. Computer Science Education
24. Marketing

Isolated Departments: Counselor Education, Education Human Services, Education: Student Teaching, Geography, Library Science, Orientation, Philosophy, Professional Studies

Appendix O

The Departmental Curriculum Prerequisite Network for Bemidji State University



Note: Departments in the indicated clusters, ordered by cluster size.

1. Biochemistry, Cellular & Molecular Biology, Biology, Chemistry, Environmental Studies, Geography, Geology, Health
2. Computer Science, Mathematics, Physics, Sociology, Statistics, Technology, Art and Design -Technology
3. Accounting, Business Administration, Physical Education, Economics, English
4. Professional Education, Modern Languages, Special Education, Spanish
5. Social Work, Political Science, Psychology
6. Philosophy, Leadership
7. Criminal Justice
8. Mass Communications
9. Ojibwe

Isolated Departments: Anthropology, Art History, Gender and Women's Studies, Humanities, Music, Science