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Rankings of MMA Fighters

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A Thesis Presented in Partial Fulfillment of the

Requirements for the Degree of

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In

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Department of Mathematics and Statistics

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Abstract

Ranking is an essential process that allows sporting authorities to determine the relative performance of athletes. While ranking is straightforward in some sports, it is more complicated in MMA (mixed martial arts), where competition is often fragmented. This paper describes the mathematics behind four existing ranking algorithms: Elo's System, Massey's Method, Colley's Method, and Google's PageRank, and shows how to adapt them to rank MMA fighters in the UFC (Ultimate Fighting Championship). We also provide a performance analysis for each ranking method.

1 Introduction

On November 14th, 2015, in UFC 193, Ronda Rousey faced Holly Holm for the title of UFC (Ultimate Fighting Championship) Bantamweight Champion. The defending champion and huge favorite, Rousey was ranked number one by official UFC rankings. Holm, while also a highly decorated fighter, was ranked seventh. In one of the most significant upsets in UFC history, Holm prevailed in a dominating fashion — a knockout head kick in the second round.

While Rousey vs. Holm was a stunning upset — it was not an isolated incident. The following year at UFC 199, the number four ranked fighter Michael Bisping knocked out the number one ranked fighter Luke Rockhold in the first round.

Although ranking fighters is not a straightforward task, the official UFC rankings are notorious for being biased. UFC rankings for fighters are generated through a voting panel of MMA (Mixed Martial Arts) media members from 21 outlets, including FightNews, Fight Network, and Top Turtle Podcast. (See [10]) The flaws in this system have motivated the creation of third-party ranking systems like FightMatrix (See [3]) — a ranking system that uses mathematics rather than opinion to rank fighters. The purpose of this paper is to explore several ranking algorithms and their potential for ranking MMA fighters.

[1]

[1] A *ranking* of items is a rank-ordered list of the items. A *rating* of items assigns a numerical score to each item. The rating is used to construct the ranking.

1.1 Elo's System

Arpad Elo (1902 - 1993) was a Hungarian-born physics professor and avid chess player (See [6]). He devised a ranking system to rank chess players. In its current form, it works as follows.

Each player starts with an initial rating[1]. Each time players i and j play against each other, their respective prior ratings $r_i^{(old)}$ and $r_j^{(old)}$ are updated to become $r_i^{(new)}$ and $r_j^{(new)}$. The formula for updating the players' ratings are,

$$r_i^{(new)} = r_i^{(old)} + K(S_{ij} - \mu_{ij}) \quad \text{and} \quad r_j^{(new)} = r_j^{(old)} + K(S_{ji} - \mu_{ji}),$$

where,

$$S_{ij} = \begin{cases} 1 & \text{if } i \text{ beats } j, \\ 0 & \text{if } i \text{ loses to } j, \\ 1/2 & \text{if } i \text{ and } j \text{ tie,} \end{cases}$$

$$\mu_{ij} = \frac{1}{1 + 10^{-d_{ij}/400}}, \text{ where } d_{ij} = r_i^{(old)} - r_j^{(old)},$$

and K, known as the "K-factor", is some defined constant. Different chess groups use different values of K. Its purpose is to properly balance the deviation between actual and expected scores against prior ratings. A higher K value results in more volatile ratings. In chess, the K-factor is allowed to change with the level of competition. For example, the International Chess Federation sets,

$$\begin{cases} K = 25 & \text{for new players until 30 recognized games have been completed;} \\ K = 15 & \text{for players with} > 30 \text{ games whose rating has never exceed 2400;} \\ K = 10 & \text{for players having reached at least 2400 at some point in the past.} \end{cases}$$

1.2 Massey's Method

In 1997, Kenneth Massey proposed a ranking system that used a least-squares approach to rank college football teams. (See [6, 8].) This method is based on the premise that the difference between two teams' ratings should predict the point differential in a game between those two teams. For instance, consider Table 1 of hypothetical score data from a round-robin tournament played by teams 1, 2, 3, and 4. Let,

$$y_k = r_i - r_j,$$

where r_i and r_j are the scores for teams i and j, respectively. The term y_k is called the *margin of victory* for game k.

Table 1: Score Data

	1	2	3	4
1		6-1	6-4	9-3
2	1-6		3-10	9-2
3	4-6	10-3		2-4
4	3-9	2-9	4-2	

From Table 1, we can form the following system of equations:

$$\begin{bmatrix} r_1 - r_2 \\ r_1 - r_3 \\ r_1 - r_4 \\ r_2 - r_3 \\ r_2 - r_4 \\ r_3 - r_4 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \\ 6 \\ -7 \\ 7 \\ -2 \end{bmatrix},$$

which translates to

$$\begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \\ 6 \\ -7 \\ 7 \\ -2 \end{bmatrix}.$$

Generally, if we have n teams, these equations can be written in the following matrix-vector form:

$$Xr = y.$$

Typically, this turns out to be an over-determined system. A least squares solution can be obtained from the normal equations,

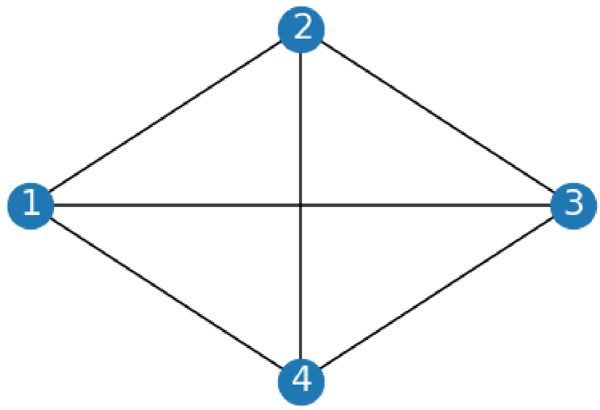
$$X^T X r = X^T y.$$

In our example, this yields,

$$\begin{bmatrix} 3 & -1 & -1 & -1 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ -1 & -1 & -1 & 3 \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \end{bmatrix} = \begin{bmatrix} 13 \\ -5 \\ 3 \\ -11 \end{bmatrix}.$$

We call $X^T X$ a new matrix M and assign p to $X^T y$, an $n \times 1$ column vector of total point differentials.

Remark The matrix $M = X^T X$ is equivalent to the Laplacian of a corresponding undirected graph, describing a fight network.



Theorem ([9, p. 147]) The rank of a Laplacian matrix of a graph G is $n - \omega(G)$ where n is the number of nodes and $\omega(G)$ is the number of connected components.

Since M is a Laplacian matrix, by Theorem 1.1.1, the rank of M is the number of nodes of a corresponding undirected graph minus the number of components. This means that, due to rank deficiency, the system $Mr = p$ will be under-determined. We can see this if we multiply the left and right-hand side of our equation by

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix},$$

to obtain

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & -1 & -1 & -1 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ -1 & -1 & -1 & 3 \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 13 \\ -5 \\ 3 \\ -11 \end{bmatrix},$$

$$\begin{bmatrix} 3 & -1 & -1 & -1 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \end{bmatrix} = \begin{bmatrix} 13 \\ -5 \\ 3 \\ 0 \end{bmatrix}.$$

If the graph is fully connected, that is, there is only one component, the rank of the $n \times n$ matrix M will be $n - 1$. To attain a full rank, we must impose one additional condition. Massey suggests adding the constraint that the rating vector r must be zero-sum. He accomplishes this by replacing the last row of M by 1's and the corresponding entry in p by 0. In our example, the system becomes,

$$\begin{bmatrix} 3 & -1 & -1 & -1 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \end{bmatrix} = \begin{bmatrix} 13 \\ -5 \\ 3 \\ 0 \end{bmatrix}.$$

Now, solving for r, we find,

$$r = \begin{bmatrix} 13/4 \\ -5/4 \\ 3/4 \\ -11/4 \end{bmatrix}.$$

From this rating vector, we construct the corresponding ranking vector,

$$R = \begin{bmatrix} 1 \\ 3 \\ 2 \\ 4 \end{bmatrix}.$$

1.3 Colley's Method

In 2001, Dr. Wesley Colley, an astrophysicist by training, put forward a new sports ranking system. His method is a modification of a straightforward ranking system based on winning percentage. (See [2, 6].)

1.3.1 Winning Percentage

Let t_i be the number of teams team i has played and let w_i be the number of times team i has won. Using the winning percentage, the rating for team i is

$$r_i = \frac{w_i}{t_i}.$$

Some disadvantages of this winning percentage method are:

- Ties in ratings occur often
- The strength of the opponent is not factored into the analysis
- Before any games are played, the rating for each team is 0/0
- A team without wins has a rating of 0

1.3.2 Laplace's Rule of Succession

Laplace's rule of succession states that if an event has occurred $p + q$ times in succession where p is the number of successes and q is the number of failures, then the probability of success on the next trial is $(p+1)/(p+q+2)$. (See [7]). If team i plays a total of $t_i = p+q$ games and wins $w_i = p$ games (and loses $l_i =$ q games), then Laplace's rule of succession says that the probability of winning the next game is,

$$\frac{1 + w_i}{2 + t_i}.$$

Based on this probability, Colley modifies the rating of team i to be,

$$r_i = \frac{1 + w_i}{2 + t_i}.$$

This suggests that a team's rating is their probability of winning the next game based on the prior outcomes. This adjustment has several advantages over the winning percentage method. Now, observe that

$$\begin{aligned} w_i &= \frac{w_i - l_i}{2} + \frac{w_i + l_i}{2} \\ &= \frac{w_i - l_i}{2} + \frac{t_i}{2} \\ &= \frac{w_i - l_i}{2} + \sum_{k=1}^{t_i} \frac{1}{2}. \end{aligned}$$

Since all teams begin with $r_k = 1/2$ and the ratings are distributed around this number, we have,

$$\sum_{k=1}^{t_i} \frac{1}{2} \approx \sum_{k \in O_i} r_k,$$

where O_i is the set of teams that have played team i. It follows that the ratings approximately satisfy the system of n equations

$$w_i = \frac{w_i - l_i}{2} + \sum_{k \in O_i} r_k.$$

Then

$$r_i = \frac{1 + w_i}{2 + t_i} = \frac{1 + \left(\frac{w_i - l_i}{2} + \sum_{k \in O_i} r_k \right)}{2 + t_i},$$

Multiplying by $2 + t_i$ gives us

$$(2+t_i)r_i = 1 + \left(\frac{w_i - l_i}{2} + \sum_{k\in O_i} r_k \right).$$

and by subtracting $\sum_{k\in O_i} r_k$ we have

$$(2+t_i)r_i - \sum_{k\in O_i} r_k = 1 + \frac{w_i - l_i}{2}.$$

This can be compactly written in the matrix-vector form, $Cr = b$, where,

$$C_{ij} = \begin{cases} 2+t_i & i=j, \\ -n_{ij} & i \neq j, \end{cases} \qquad , \qquad b_i = 1 + \frac{1}{2}(w_i - l_i),$$

where n_{ij} is the number of times teams i and j played each other.

1.3.3 Cholesky Factorization

A symmetric matrix A is positive definite if $x^T Ax$ is positive for all non-zero vectors x. A positive definite matrix A can be expressed as $A = X^T X$ for a non-singular matrix X where X is upper triangular with positive diagonal elements. It can be shown that the Colley matrix, C, is a real symmetric positive definite. This means that we can efficiently solve the system $Cr = b$ using Cholesky factorization. (See [5])

Remark The Massey and Colley ranking systems are related by the formula $C = 2I + M$. This allows us to easily construct C having already constructed M.

1.4 PageRank

Initially developed by Google co-founders Larry Page and Sergey Brin to rank web pages, PageRank is perhaps the most noteworthy ranking algorithm of the 21st century. To describe the PageRank algorithm, we begin by considering the directed graph shown in Figure 1. (See [1])

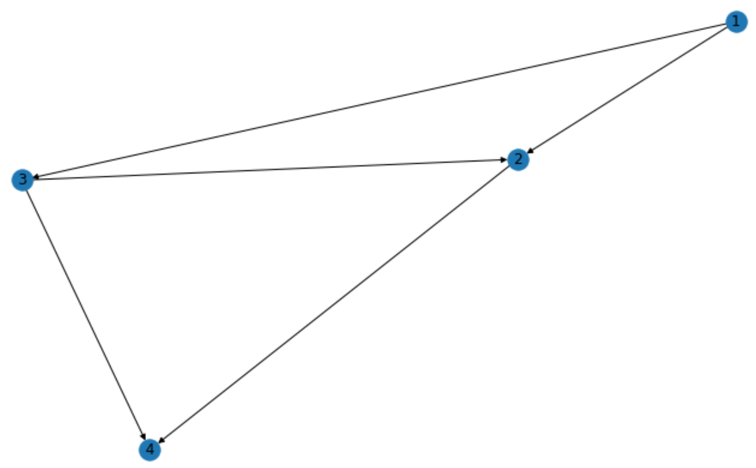


Figure 1: A directed graph with vertex set $V = \{1, 2, 3, 4\}$ and edge set $E = \{(1, 2), (1, 3), (2, 4), (3, 2), (3, 4)\}$

Remark The adjacency matrix $A = (a_{ij})$ of a graph $G := (V, E)$ with $|V|$ vertices and $|E|$ edges is,

$$a_{ij} = \begin{cases} 1, & \text{if } (v_i, v_j) \in E, \\ 0, & \text{otherwise }. \end{cases}$$

From the directed graph in Figure 2, can form the adjacency matrix,

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

We then divide each row of matrix A by the number of out-degrees per page to produce the hyperlink matrix, H.

$$H = \begin{bmatrix} 0 & 1/2 & 1/2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1/2 & 0 & 1/2 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Because PageRank requires a stochastic matrix in which all the row sums are equal to 1, any row that contains all zeros has all elements replaced by $1/|V|$.

$$H = \begin{bmatrix} 0 & 1/2 & 1/2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1/2 & 0 & 1/2 \\ 1/4 & 1/4 & 1/4 & 1/4 \end{bmatrix}.$$

Finally, to ensure that the graph is strongly inter-connected and irreducible, the matrix is modified to obtain the Google matrix,

$$G = \alpha H + (1 - \alpha)E,$$

where α, called a *damping factor*, is set as $\alpha = 0.85$ and E is a $|V| \times |V|$ matrix entirely populated by $1/|V|$. Note that H and G are row-stochastic, and thus can be viewed as transition matricices of Markov chains.

$$G = \begin{bmatrix} 0.0375 & 0.4625 & 0.4625 & 0.0375 \\ 0.0375 & 0.0375 & 0.0375 & 0.8875 \\ 0.0375 & 0.4625 & 0.0375 & 0.4625 \\ 0.25 & 0.25 & 0.25 & 0.25 \end{bmatrix}.$$

The final rating vector is then computed using the power method,

$$r_k = r_0 G^k$$

where r_0 is an initial estimated rating vector which we choose to populate with values chosen randomly over $[0, 1)$.

2 Ranking Fighters

2.1 Data

The data used for analysis comes from the Kaggle data set *UFC-Fight historical data from 1993 to 2021*, which was scraped from the official UFC site, ufcstats.com (See [11]). For all algorithms, we use data up to 2020 to test the ranking system's ability to predict the outcomes of the 451 fights with an outcome in 2020.

2.2 Applying Elo's System

Each fighter has a rating. By convention, each fighter will start with a rating of 1500. When two fighters face each other, they wager a portion of their rating. The K-factor determines the size of the portion they wager. We establish this K-factor experimentally by choosing the value that yields the highest number of successfully predicted fight outcomes.

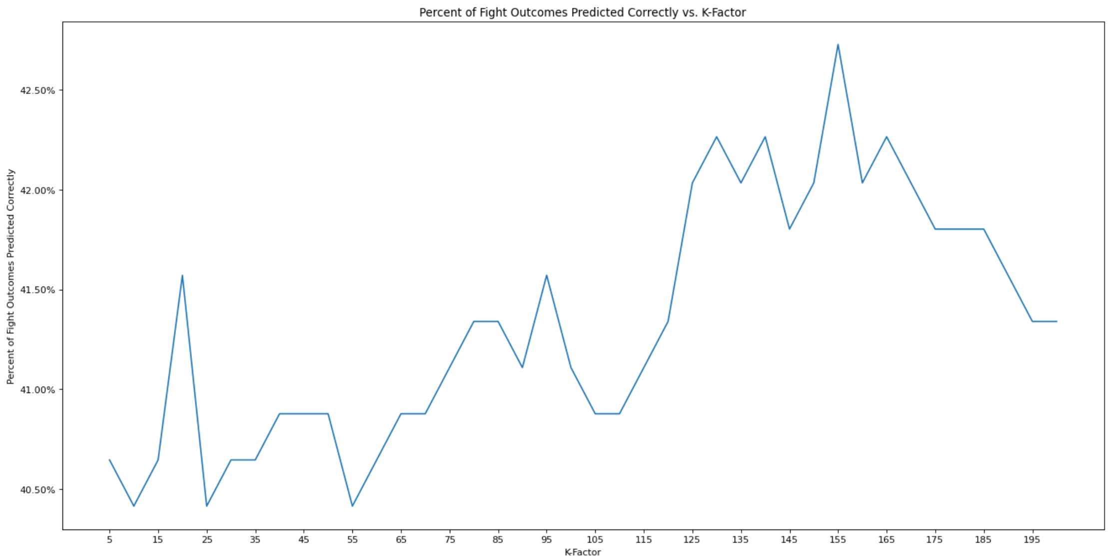


Figure 2: Percent of Fight Outcomes Predicted Correctly vs K-Factor using Elo's System

Based on this, we choose our K-factor to be 155.

2.3 Applying Massey and Colley Methods

Combat sports such as MMA or boxing present a unique challenge when ranking fighters. They are:

(i) Many fighters will never face each other. In the Massey and Colley methods, this will produce under-determined linear systems.

(ii) Fighters rarely have equal numbers of bouts. One fighter may have five matches in a year, while another only has one. Although the fighter with more fights may have more wins, the Massey or Colley systems may rank the fighter with fewer fights higher.

(iii) Massey's Method requires defining a margin of victory. In the

UFC, there is no defined point system like in football or basketball, so there is no clear way to define this margin of victory.

The first two challenges can be remedied by the introduction of a *super user* (or *super fighter*) (See [4]). This super fighter is a hypothetical fighter that beats every fighter they face. By introducing this super fighter, we can guarantee that our network of fighters is connected. Further, this fictitious fighter can reduce the effect caused by an unequal number of bouts.

The third challenge remains. We would like to implement a formula that is simple and minimizes ad-hoc decisions. For Massey's Method, we choose to define the margin of victory by $m/(m+t)$, where t is the time the bout lasts in minutes and m is a constant set as the maximum theoretical fight time in minutes. In the UFC, a usual match has 3 rounds each lasting 5 minutes. This gives a minimum margin of victory of $15/(15+15) = 1/2$ and a maximum margin of victory of $15/(15+0) = 1$. By defining the margin of victory in this way, we assure that the margin of victory is positive, a greater margin of victory is awarded to fighters who win their bouts faster, and the hypothetical greatest margin of victory is twice that of the hypothetical smallest margin.

2.3.1 Minimum Cutoff

It may be necessary to introduce a super fighter to remedy a disconnected network of fighters; however, the super fighter doesn't need to face everybody. Changing whom the super fighter faces can drastically change the rankings of fighters. To correct the inflated ratings of

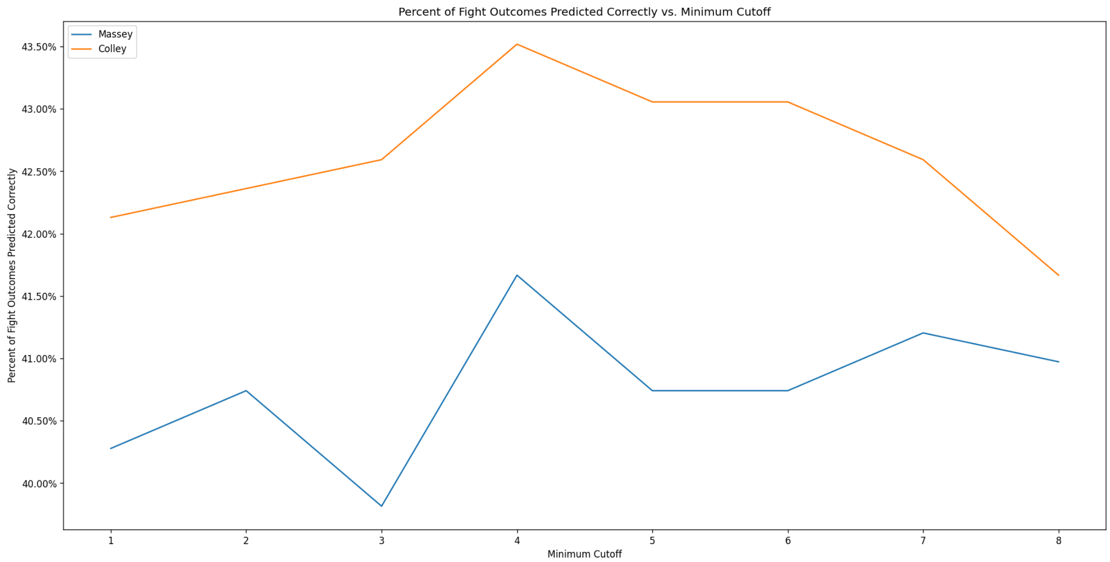


Figure 3: Percent of Fights Predicted Correctly vs. Minimum Cutoff

fighters with few bouts, we choose to have the super fighter only face fighters that have had a number of bouts below some minimum cutoff. We establish this minimum cutoff value experimentally by choosing the value that maximizes the percent of fight outcomes predicted correctly in 2020. Based on our results (See Figure 3), we decide to have the super fighter face fighters with less than or equal to four bouts.

2.4 Computation

We now return to our introduction and rank all the Women's Bantamweight fighters before UFC 193. As demonstrated in Figure 4, there is a significant disparity in the number of bouts among fighters.

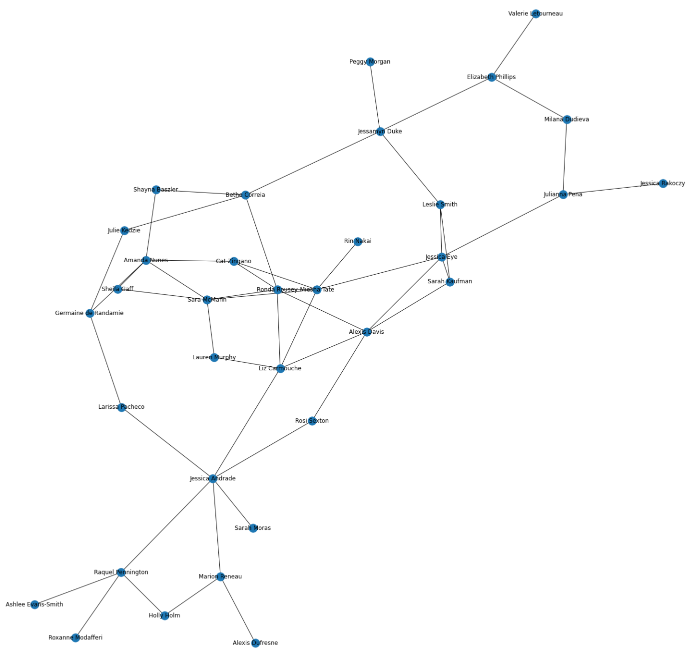


Figure 4: Network of UFC Women's Bantamweight Fights (Feb. 23, 2013, - Nov. 9, 2015)

After introducing the super fighter, we obtain the network shown in Figure 5. Table 2 shows the official UFC rankings published in November 9th 2015, and the Colley, Massey, Elo, and PageRank rankings using data up to November 9th, 2015,

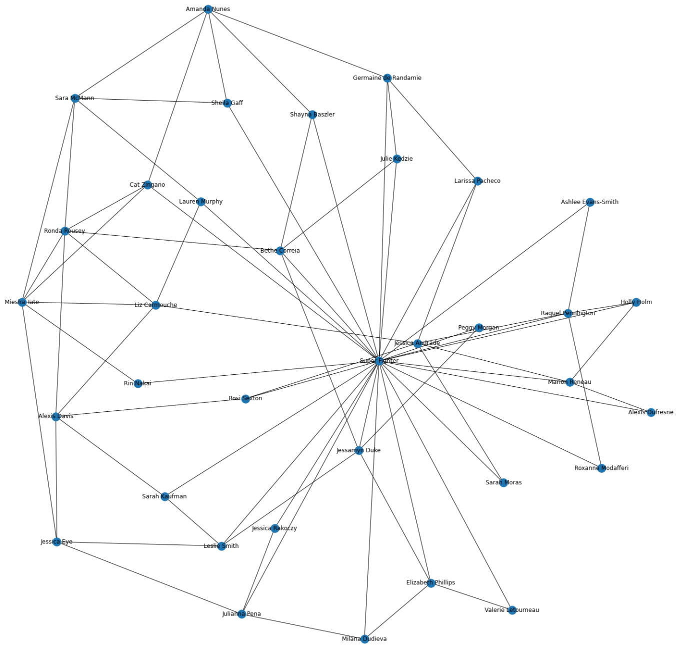


Figure 5: After Introducing the Super Fighter

Table 2: Ranking of UFC Women's Bantamweight Fighters (November 9th, 2015)

Official UFC Rankings	Adapated Colley Rankings	Adapated Massey Rankings	Elo Rankings	PageRank
Ronda Rousey	Ronda Rousey	Ronda Rousey	Ronda Rousey	Ronda Rousey
Cat Zingano	Cat Zingano	Cat Zingano	Julianna Pena	Cat Zingano
Amanda Nunes	Julianna Pena	Julianna Pena	Alexis Davis	Jessica Andrade
Sarah Kaufman	Alexis Davis	Amanda Nunes	Amanda Nunes	Holly Holm
Julianna Pena	Amanda Nunes	Miesha Tate	Holly Holm	Miesha Tate
Sara McMann	Holly Holm	Alexis Davis	Cat Zingano	Alexis Davis
Holly Holm	Miesha Tate	Holly Holm	Bethe Correia	Raquel Pennington
Bethe Correia	Bethe Correia	Marion Reneau	Marion Reneau	Amanda Nunes
Jessica Eye	Marion Reneau	Liz Carmouche	Valerie Letourneau	Marion Reneau
Liz Carmouche	Raquel Pennington	Jessica Eye	Raquel Pennington	Julianna Pena
Marion Reneau	Germaine de Randamie	Sara McMann	Germaine de Randamie	Liz Carmouche
Raquel Pennington	Valeria Letourneau	Bethe Correia	Miesha Tate	Bethe Correia

It is interesting to note that while Rousey is ranked first in all of the rankings, Holm is ranked higher in the Colley, Elo, and PageRank rankings than in the UFC rankings. Also, despite being ranked fourth by the UFC, Sarah Kaufman does not appear in the top 12 rankings of any of the methods.

Repeating the process with Men's Middleweight fighters before UFC 199, we find the following.

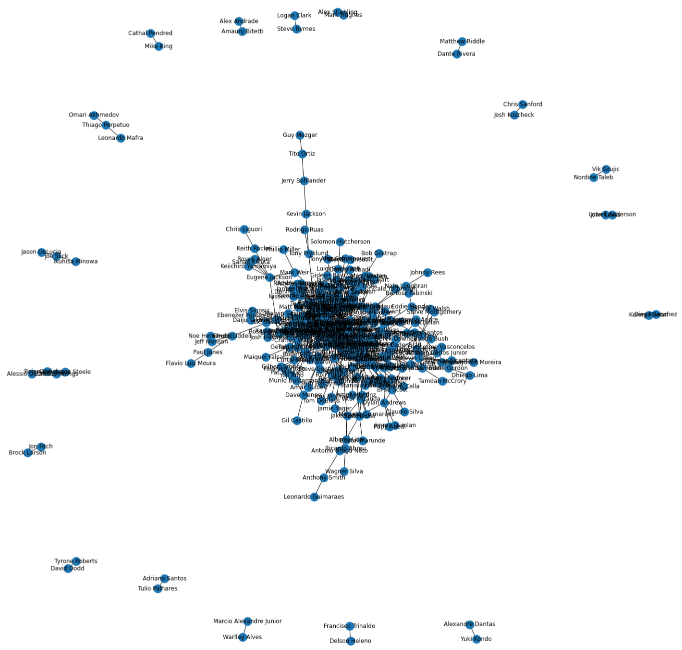


Figure 6: Network of UFC Men's Middleweight Fights (July 27, 1997, - May 31, 2016)

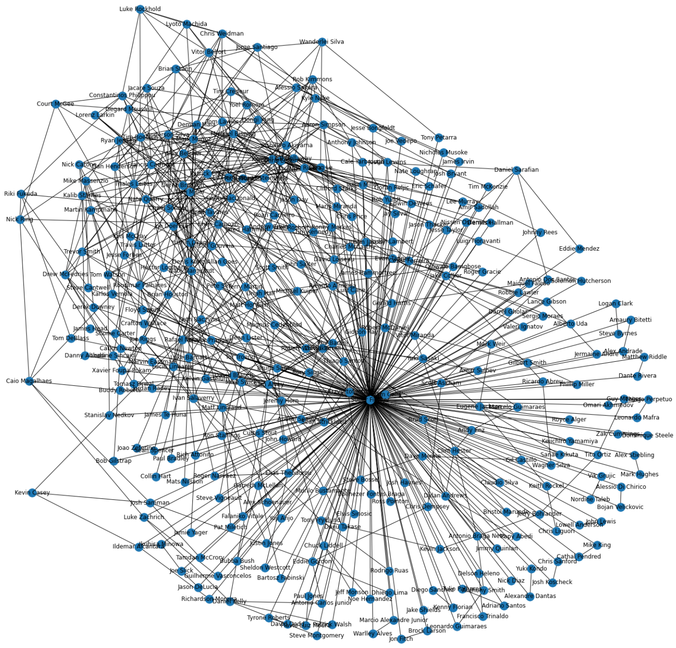


Figure 7: After Introducing the Super Fighter

Table 3: Ranking of UFC Men's Middleweight Fighters (May 31, 2016)

Official UFC Rankings	Adapted Colley Rankings	Adapted Massey Rankings	Elo Rankings	PageRank
Luke Rockhold	Yoel Romero	Ronaldo Souza	Anderson Silva	Vitor Belfort
Chris Weidman	Chris Weidman	Yoel Romero	Yoel Romero	Luke Rockhold
Ronaldo Souza	Ronaldo Souza	Luke Rockhold	Chris Weidman	Chris Weidman
Vitor Belfort	Anderson Silva	Chris Weidman	Demian Maia	Yoel Romero
Michael Bisping	Luke Rockhold	Robert Whittaker	Ronaldo Souza	Anderson Silva
Anderson Silva	Robert Whittaker	Derek Brunson	Robert Whittaker	Michael Bisping
Robert Whittaker	Vitor Belfort	Lyoto Machida	Derek Brunson	Ronaldo Souza
Lyoto Machida	Derek Brunson	Gegard Mousasi	Brad Tavares	Thales Leites
Gegard Mousasi	Tim Kennedy	Vitor Belfort	Vitor Belfort	Demian Maia
Tim Kennedy	Michael Bisping	Tim Kennedy	Tim Kennedy	Gegard Mousasi
Uriah Hall	Demian Maia	Anderson Silva	Michael Bisping	Dan Henderson
Derek Brunson	Lyoto Machida	Thiago Santos	Krzysztof Jotko	Tim Boetsch

2.4.1 Spearman's Rank Correlation

Spearman's rank correlation coefficient, R_s, is a statistical measure of the strength of a link or relationship between two data sets. We can use it to quantify the similarity between two rankings. The coefficient can hold values from -1 to 1. A value of 1 indicates a perfect positive association between rankings, a value of 0 indicates no association between rankings and a value of -1 indicates a perfect negative association between rankings. Table 3 reports the rank correlation between each pair of rankings in Table 2 and an associated p-value where the null hypothesis is that there is no relationship between rankings. We compute the pairwise rank correlations below.

Table 4: Spearman's Rank Correlation (Top 12 Women's Bantamweight Rankings)

	R_s	p
UFC - Colley	0.3603	0.20
UFC - Massey	0.3143	0.50
UFC - Elo	0.243	0.50
UFC - PageRank	0.2937	0.50
Colley - Massey	0.9161	0.001
Colley - Elo	0.8042	0.005
Colley - PageRank	0.4283	0.20
Massey - Elo	0.8252	0.002
Massey - PageRank	0.3112	0.50
Elo - PageRank	0.2535	0.50

Table 5: Spearman's Rank Correlation (Top 12 Mens's Middleweight Rankings)

	R_s	p
UFC - Colley	0.4126	0.20
UFC - Massey	0.2587	0.50
UFC - Elo	-0.0227	0.50
UFC - PageRank	0.5944	0.05
Colley - Massey	0.6224	0.05
Colley - Elo	0.4878	0.20
Colley - PageRank	0.6818	0.02
Massey - Elo	0.3147	0.50
Massey - PageRank	0.3147	0.50
Elo - PageRank	0.4545	0.20

2.4.2 Performance Analysis

In 2020, there were 451 UFC fights with a determined winner. A ranking correctly predicted the outcome if, prior to the outcome of a match, the winner is ranked higher than the loser. The results for each ranking method are summarized in Table 6.

Table 6: Percent of Fights in 2020 Predicted Correctly

	Percent of Fights Predicted Correctly
Adapted Colley's Method	44.54 %
Adapted Massey's Method	42.65%
Elo's System	43.84 %
PageRank	42.89 %

We can see that all of the methods performed similarly with the Adapated Colley's Method performing slightly above the rest.

3 Conclusion

Ranking is an essential process that allows sporting authorities to determine the relative performance of athletes. While ranking is straightforward in some sports, it is more complicated in MMA (mixed martial arts), where competition is often fragmented. We have described the mathematics behind four existing ranking algorithms: Elo's System, Massey's Method, Colley's Method, and Google's PageRank, and shown how to adapt them to rank MMA fighters in the UFC (Ultimate Fighting Championship). While none of the methods performed well in predicting future outcomes, (the best performing method, Colley's Method, only predicted 44% of outcomes correctly) all of the rankings still have use in matchmaking or for personal interest. Any of the methods described here could be easily extended to ranking other sports where, like MMA, competition is fragmented.

A future research topic could be investigating the efficacy of two chess ranking algorithms, Glicko and Glicko-2, in ranking MMA fighters. Further, we could investigate modifying point differential formula used in Massey's method to achieve a better outcome.

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4 Appendix

4.1 Python Code

4.1.1 Colley and Massey Rankings

```
import pandas as pd
import numpy as np
import scipy
import networkx as nx
import datetime as dt

# Dates
start_day =  <startdate>
end_day = <enddate>
data = data_raw[data_raw['date'].between(start_day, end_day)]

# Weightclass
data_mw = data.loc[data['Fight_type'].str.contains(<weightclass>)]

# Plot
fighters = data_mw[['R_fighter', 'B_fighter']]

G = nx.from_pandas_edgelist(fighters, 'R_fighter', 'B_fighter')
from matplotlib.pyplot import figure
figure(figsize=(20, 20))
nx.draw(G, with_labels=True)


# Get new column minutes
minutes = data_mw['last_round_time'].str.split(':').\
```

```
apply(lambda x: int(x[0]) + int(x[1])/60)
data_mw.insert(4, 'minutes', minutes)
fight_time = 5*(data_mw['last_round'] - 1) + data_mw['minutes']
data_mw.insert(4, 'fight_time', fight_time)

# Remove bouts without a winner
data_mw = data_mw[data_mw['Winner'].notna()]

# Get average fight time
avgtime = data_mw['fight_time'].mean()

# Add super fighter

minimum_cutoff = 4

degrees = [val for (node, val) in G.degree()]
fighters = [node for (node, val) in G.degree()]
max_degree = max(degrees)
min_degree = min(degrees)

# remove if deg is small
remove = [node for node,degree in\
          dict(G.degree()).items() if degree < 4]
G.remove_nodes_from(remove)

while min(degrees) <= minimum_cutoff:
    players_few_fights = []
    for i in range(len(degrees)):
        if degrees[i] <= minimum_cutoff:
            players_few_fights.append(fighters[i])
```

```
            degrees[i] = degrees[i] + 1

    for i in range(len(players_few_fights)):
        new_row = ['Super Fighter', players_few_fights[i], ' ', ' ',
                   avgtime, ' ', ' ', end_day + \
                   ' 00:00:00', ' ', 'Super Fighter' ]
        data_mw = data_mw.append(pd.Series(new_row,
                index=data_mw.columns[:len(new_row)]), ignore_index=True)

# Plot network after adding super fighter

G = nx.from_pandas_edgelist(data_mw, 'R_fighter', 'B_fighter')
from matplotlib.pyplot import figure
figure(figsize=(20, 20))
nx.draw(G, with_labels=True)
```

4.1.2 PageRank Rankings

```
def pagerank(M, num_iterations: int = 100, d: float = 0.85):
    N = M.shape[1]
    v = np.random.rand(N, 1)
    v = v / np.linalg.norm(v, 1)
    M_hat = (d * M + (1 - d) / N)
    for i in range(num_iterations):
        v = M_hat @ v
    return v

PR = nx.from_pandas_edgelist(data_mw, source = 'Loser', target \
                             = 'Winner', create_using=nx.DiGraph())
pr_adj = scipy.sparse.csr_matrix.toarray\
(nx.adjacency_matrix(PR)).astype('float64')
```

```
m, n = pr_adj.shape
pr_adj_copy = pr_adj.copy()
for i in range(n):
    for j in range(m):
        if pr_adj[i, j] != 0:
            pr_adj[i, j] = pr_adj[i, j]/sum(pr_adj_copy[:,j])

for i in range(n):
    if sum(pr_adj[:,i]) == 0:
        pr_adj[:,i] = np.full(n, 1/n)

pr_r = pagerank(pr_adj, 7500, 0.85)
argsort_pr = np.ndarray.flatten(np.argsort(pr_r, axis=0))

nodes = list(PR.nodes())
ranked_fighters_pr = []
for i in argsort_pr:
    #print(i)
    ranked_fighters_pr.append(nodes[i])

ranked_fighters_pr
```

.

4.1.3 Elo Rankings

```
fighters = data_mw[['R_fighter', 'B_fighter']]

G = nx.from_pandas_edgelist(fighters, 'R_fighter', 'B_fighter')
fighters = [node for (node, val) in G.degree()]

#Initial Ratings Vector
```

```
ratings = np.full(len(fighters), 1000)

losers = []

for j in range(len(data_mw)):
    if data_mw['Winner'].values.tolist()[j] == \
    data_mw['R_fighter'].values.tolist()[j]:
        losers.append(data_mw['B_fighter'].values.tolist()[j])
    else:
        losers.append(data_mw['R_fighter'].values.tolist()[j])

data_mw.insert(8, 'Loser', losers)
data_mw = data_mw[data_mw['Winner'].notna()]
data_mw = data_mw[data_mw['Loser'].notna()]



elo_mw = data_mw[['Winner', 'Loser']]
winnerloser = elo_mw.values.tolist()

def elo(r_winner, r_loser):
    K = 155
    S_win = 1
    S_lose = 0
    d_win = r_winner - r_loser
    d_lose = r_loser - r_winner
    mu_win = 1/(1+10**(-d_win/400))
    mu_lose = 1/(1+10**(-d_lose/400))
    r_winner = r_winner + K*(S_win - mu_win)
    r_loser = r_loser + K*(S_lose - mu_lose)
    return r_winner, r_loser
```

```
for i in range(len(winnerloser)):

    winner = winnerloser[i][0]
    loser = winnerloser[i][1]

    winner_pos = fighters.index(winner)
    loser_pos = fighters.index(loser)

    r_winner = ratings[winner_pos]
    r_loser = ratings[loser_pos]

    r_winner_new, r_loser_new = elo(r_winner, r_loser)

    ratings[winner_pos] = r_winner_new
    ratings[loser_pos] = r_loser_new

argsort_elo = np.ndarray.flatten(np.argsort(-1*ratings, axis=0))

ranked_fighters_elo = []
for i in argsort_elo:
    #print(i)
    ranked_fighters_elo.append(fighters[i])

ranked_fighters_elo
```